Robots Are Us: Some Economics of Human Replacement*

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Abstract

Will smart machines do to humans what the internal combustion engine did to horses – render them obsolete? If so, can putting people out of work or, at least, good work leave them unable to buy what smart machines produce? Our model's answer, presented as a thought piece, not a reality play, is potentially yes. New AI technology that increases today's labor demand decreases tomorrow's, creating a boom-bust cycle that can, for a range of plausible parameters, reduce the welfare of everyone alive in the long run. Carefully crafted redistribution policies can prevent such immiserating growth. But certain supposed solutions, such as limiting intellectual property rights or restricting labor supply, can make matters worse.

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1 Introduction

Whether it's bombing our enemies, steering our planes, fielding our calls, rubbing our backs, vacuuming our floors, driving our taxis, or playing Jeopardy, it's hard to think of hitherto human tasks that smart machines can't do or won't soon do. Few smart machines look remotely human. But all combine brains and brawn – artificial intelligence (AI) and physical capital. And they all share one creator – humankind. Indeed, we've taught smart machines to not only pick our brains, but to optimize the process.

Will outsourcing ourselves to smart machines deliver economic nirvana – personal genies to fulfill our every material need? Or will our redundancy leave us earning too little to buy what smart machines can produce? This paper illustrates both possibilities within a simple OLG model. Our setting features two types of workers consuming two goods for two periods. Though elemental, our mathematical parable admits a range of dynamic reactions to 'robots,' some quite unpleasant.

Our model features high- and low-tech workers. Both work full time, but only when young. There are two goods – 'corn' and 'prayer'. Corn is produced with capital and 'code'. By code, we mean rules, instructions, AI-algorithms, explicit software, etc. Code, whether implemented digitally or not, is subject to depreciation. Its current stock is the sum of the depreciated stock plus the newly produced flow. New code is produced exclusively by high-tech workers.[1] Prayers are produced by a combination of high- and low-tech labor. High-tech workers are fully mobile between the coding and praying sectors; i.e., they choose between these two tasks. Coders rent out their script in the period it's written and sell rights to its future use.

Code is measured in efficiency units, not literal lines of instructions, as less software is often more software. Old code needs to be retained, maintained, and updated. But it's otherwise a perfect substitute for new code. Hence, old code limits the demand for new code and, thus, for coders.

The potential for dead coders, whose work is embedded in existing code, to obsolesce living coders – the dead competing with the living – is illustrated by *Junior* – 2013's World Computer Chess Champion. Junior can beat all living and, arguably, all future human beings. Consequently, Junior's old code has put chess programmers out of business at least in beating humans at chess. Junior is, of course, a very smart machine – a robot that combines capital and code with a human attendant to plug him in.

Large language models, such as the one powering ChatGPT, also have the potential to supplant human workers in a vast array of tasks (Eloundou, Manning, Mishkin and Rock 2023). Many of these technologies

[1] The Bureau of Labor Statistics (2017) attributes 5.25 percent of 2016 U.S. wages to those in computer and mathematical occupations. But our concept of coders is far broader.

are trained via unsupervised machine learning that processes massive volumes of human communications. Their knowledge ingestion is often fine-tuned by humans via so called supervised learning. Thus, all robots, be they Junior or ChatGPT, are, effectively, us.

We capture the ability of old code to compete with new coders via a code retention rate, δ. Our interest is the economic impact of a rise in δ due, say, from the invention of the silicon chip or deep-learning architecture. The obverse of code retention is code depreciation $(1-\delta)$. Code depreciation captures the need to maintain and update code, including recalibrating or retraining smart machines to current economic, technological and market needs and conditions. And it provides an ongoing demand for coders. If conditions change slowly or the automated system can recalibrate itself, with unsupervised or reinforcement learning, that demand may be low. This is captured by a high δ.

The response, in our model, to a rise in the code retention rate is a tech boom that raises the demand for new code and coders. The resulting rise in high-tech and, indeed, low-tech workers' earnings engenders more national saving and capital formation, reinforcing the boom. But over time, as the stock of legacy code grows, the demand for new code and, thus, coders falls. Labor's share of national income declines and more coders move into prayer. Depending on the substitutabilty or complementarity of high- and low-tech workers in providing prayers, low-tech workers can see their wages fall or rise.

The eventual decline in wages of high- and, potentially, low-tech workers limits what the young save and invest. This means less physical capital available for future use. If the capital stock falls by enough, which arises for a wide range of empirically plausible parameter values, the economy ends up producing less output notwithstanding its improved technology. In this case, increased robotization, as modeled here, can leave everyone worse off, i.e., it can be immiserating. This outcome represents a dynamic refutation of Say's Law as extra short-run supply of one input (code) reduces the long-run demand for, and thus supply of, another (capital). In this sense, supply destroys its own demand. This process holds even for the wide range of parameter-dependent equilibria in which higher code retention leaves the economy, on balance, better off if not far better off.

Of course, demographics, productivity growth, and other major forces will also shape the global economy's future. These forces may preclude technological immiseration in settings when it would otherwise arise. But the impetus toward immiseration posited here would, under the right conditions, nonetheless, limit increases in long-run welfare, output, and wages.

Our simulations find reflection in the data. Karabarbounis and Neiman (2014) document a decline in labor's income share. Certainly, the tech sector has experienced booms and busts in recent decades.[2] The U.S. net national saving rate, properly measured, averaged 13 percent in the fifties and sixties. It's since steadily declined to roughly 3 percent – the average over the past dozen years. In line with (Feldstein and Horioka 1980), the nation's net domestic investment rate has also declined, but not as rapidly. This has, of course, meant lowered growth in capital per worker. As shown in (Dobrescu, Kotlikoff and Motta 2012), other advanced nations, including France, Germany, Italy, Spain, and the UK have experienced similar dramatic postwar declines in net national saving rates. As for the return to capital, it has risen over the post war notwithstanding a major decline in real yields on government bonds pre COVID.[3]

Our model's ability to produce a boom-bust cycle in wages and welfare hinges on its OLG structure, particularly the intergenerational redistribution arising from the tech-induced business cycle. Such redistribution is absent in single-agent models. Indeed, capital shortages in such models lead to higher interest rates, which induce greater saving and capital formation. [4]

Our main findings assume that code is excludable and rival in its use. This is a reasonable approximation if software needs to be specialized to each application. Evidence indicates that firm-specific intangible investments in creating 'code assets' – necessary to take advantage of fully non-rival software innovations – are ten times larger than measured purchases of hardware and software (Brynjolfsson, Rock and Syverson 2018). In the case of Large Language Models, this specialization takes the form of 'fine-tuning', in which supervised and reinforcement learning are used to adapt a general AI system to a specific context.

We also consider the implications of relaxing these assumptions. If the same code can not only be copied for free, but can also be costlessly adapted to multiple applications, it resembles a public good. After considering excludable and rival code, we consider cases in which code is non-rival and, potentially, non-excludable. This lets us evaluate different intellectual property (IP) regimes, which may be of interest to policymakers worried about the effects of emerging AI technologies on market concentration and labor demand (Benzell and Brynjolfsson 2018). We find that a rise in code retention can still immiserate the economy when code is non-rival. Surprisingly, a policy of making all code open source (non-excludable) can backfire. Doing so provides a windfall to the initial young generation, not by raising their wages when young, but by raising their asset incomes when old. This leads the initial young to consume more and save less, leading to capital's crowding out – precisely the immiseration mechanism arising when code is excludable.

[2] See https://www.businessinsider.com/tech-boom-bust-recession-2023-3

[3] Measures of the U.S. net national saving rate, net domestic investment rate, and return to capital are based on recent calculations, available upon request, using NIPA and Federal Reserve Financial Accounts data.

[4] There is strong evidence against the operative intergenerational altruism justifying the single-agent model. See, for example, Altonji, Hayashi, and Kotlikoff (1992,1997), Hayashi, Altonji and Kotlikoff (1996), and Abel and Kotlikoff (1994).

This said, open sourcing lowers lowers market concentration by encouraging more entry by small firms.

Our appendix A considers whether long-run immiseration can arise when agents have more control over technological change as in Acemoglu (2002) and Acemoglu and Restrepo (2018). In this task-model extension, firms can create either labor-substituting or capital-substituting code. We find that immiseration is still possible provided labor-substituting code technology is more advanced than capital-substituting code technology.[5]

The next section considers the long-standing concern that technical change can hurt workers and reviews a small portion of the relevant literature. Section 3 presents the model and demonstrates the potential for immiseration analytically. It concludes with a subsection demonstrating our paper's main mechanisms in a simplified version of the model. Section 4 illustrates, numerically, the model's surprising range of outcomes, including immiseration, arising from a rise in code retention. Section 5 shows that code rivalry and property rights can make a major difference. Section 6 considers evidence concerning the model's main predictions. Section 7 concludes.

2 Background and Literature Review

Concern about new technology dates at least to Ned Ludd's destruction of two stocking frames in 1779 near Leichester, England.[6]

Sixty-five years later, Marx (1867) restated Ned Ludd's warning about machines replacing humans: "Within the capitalist system all methods for raising the social productivity of labour are put into effect at the cost of the individual worker." Keynes (1933) also raised technology's potential for job destruction, writing in the midst of the Great Depression that "We are being afflicted with a new disease ..., namely technological unemployment."[7]

[5]Previous versions of this paper included an extension where firms choose between a range of production technologies. In such a model, immiseration remains possible. Additionally, Kondratiev-type business cycles, like those seen in our non-rival and non-excludable code scenario, can arise.

[6]Ludd, a weaver, was whipped for indolence before wreaking revenge on the machines. More than three decades later, in 1812, 150 armed workers – self-named Luddites – marched on a textile mill in Huddersfield, England. Their purpose was to smash equipment. The British army promptly killed 19 of the protesters. Later that year the British Parliament passed *The Destruction of Stocking Frames Act*, authorizing death for those vandalizing machines. Nonetheless, Luddite riots continued for several years, eventuating in 70 hangings.

[7]But Keynes called this "only a temporary phase of maladjustment" and predicted a future of leisure and plenty one hundred years hence. His contention that short-term pain facilitates long-term gain reinforced Schumpeter's 1942 encomium to "creative destruction".

In the fifties and sixties, with employment high and wage growth rapid, Keynes' and Schumpeter's views held sway. Those raising concerns about technology were regularly dismissed as "Luddites." But in recent years the swift loss of all manner of jobs to smart machines has led economists to rethink Luddism. Erik Brynjolfsson and Andrew McAfee (2014)'s book, *Race Against the Machine*, and Aghion, et. al.'s (2017) recent paper, *Artificial Intelligence and Economic Growth*, are just two examples of a burgeoning literature. Brynjolfsson and McAfee emphasize the ongoing role of machines in changing relative compensation across occupations, increasing inequality and decreasing labor force participation. Aghion and co-authors connect AI to the recent decline in labor's output share. But they also view AI as potentially just a new form of automation – one likely subject to *Baumol's Cost Disease*, with long-run outcomes ultimately determined not by what AI can do, but what it can't do.

The long run can take a long time. Moreover, a key message of OLG models is that where the economy ends up depends on how it gets there. Hence, the focus by Autor, Levy, and Murnane (2003), Acemoglu and Autor (2011), Autor and Dorn (2013) and others on how smart machines are impacting current employment and wage trends is well placed. Each finds significant outsourcing of middle-skilled workers by smart machines. Goos, Manning, and Salomons (2010) offer supporting evidence for Europe. Margo (2013) points to similar *labor polarization* during the early stages of America's industrial revolution. Many economists are now connecting robotization to the ongoing decline in labor's share of output. Hemous and Olson (2014) is an example. Their model has capital substituting for low-tech and complementing high-tech labor and explains trends in labor's share and income inequality since the 1960s.

The deleterious labor-market impact of smart machines is not without its skeptics. Mishel, Shierholz, and Schmitt (2013) argue that 'robots' can't explain post-1970's U.S. job polarization given the observed timing of changes in relative wages and employment. Autor (2015) is another skeptic, at least over the long term. He points out that the automobile displaced equestrian drivers but introduced myriad occupations for humans in the auto and other industries. Autor's argument applies to humans, but not to horses, who suffered massive permanent job loss. Thus, when it comes to AI, the question is whether today's humans are yesterday's horses.

Our model features an endogenous technological and growth response to an exogenous technology shock – the aforementioned rise in the rate of code retention). Hence, our study connects, to a degree, to the endogenous growth literature, whose major contributions include Schumpeter (1939), Arrow (1962), Uzawa (1965), Sidrauski (1967), Lucas (1988), Romer (1990), Rebelo (1991), Ortigueira and Santos (1997), Zeira (1998), Acemoglu (1998), Howitt (1999), Zuleta (2008), and Peretto and Seater (2013).

As for modeling automation, economists have taken a range of approaches. Zeira (1998) posits the availability, at a cost, of labor-substituting machines and shows that countries with high labor costs and low interest rates will industrialize more rapidly than others. This process produces a dispersion in global per capita income. Zuleta (2004, 2008) considers the choice not of labor-replacing machines, but of the degree of capital intensivity. As in Zeira (1998), rich economies expand relative to poor economies, which can't afford to increase their degree of capital intensity. Zeira (2004)'s model is OLG. Hence, he too finds that the decline in labor income can cause economic problems over time. In his case, it's the inability to achieve long-run growth absent the presence of bequests. (Benzell, Kotlikoff, LaGarda and Ye 2021) consider the impact of global automation, which they capture in a Zeira-type manner. Specifically, they posit region-specific Cobb-Douglas production and the endogenous choice of capital's share.

Our model differs from (Sachs and Kotlikoff 2012) and (Sachs, Benzell and LaGarda 2015). In those papers, robots, comprising capital, substitute perfectly for labor. In our model, durable capital and durable code are complements and immiseration arises, when it does, from high-tech workers producing additional code that competes with future high-tech workers.

Acemoglu (1998) features firms that invest in technology that differentially raise the productivity of their least expensive inputs. Rourke, et. al. (2013) examines 18th and 19th century technological change in England with special focus on the skill premium. His model, which is similar to Acemoglu's (1998), appears capable of matching the trend in the skill premium over the period. Peretto and Seater (2013) extend Zuleta (2008). They consider monopolistically competitive firms that invest in particular technologies depending on their relative costs. In their model, firms may specialize in the use of one technology or produce with multiple technologies. Acemoglu and Restrepo (2018) endogenize the automation of labor as well as the invention of new labor-intensive products. The former (latter) occurs to a greater (lessor) degree when wages are high (low). They show that balanced growth can arise with the demand for labor to perform innovative tasks offsetting job loss due to automation.

Many of these models, including Acemoglu and Restrepo (2018), have capital accumulation as arising from the savings of a representative agent. This means that changes in labor's share of national income do not tend to decrease the saving rate. In fact, decreases in labor's share of income increase the interest rate, and therefore, through the representative agent's inter-temporal Euler condition, increase the saving rate. This precludes the possibility of long-term immiseration due to automation-induced dissaving. If Acemoglu and Restrepo (2018) or one of the other production models were to be combined with an OLG model of households, more negative long-term outcomes would arise.

Guerreiro, Rebelo, and Teles (2017), Costinot and Werning (2018) and others have examined the use of fiscal policy to offset technology-induced redistribution. Costinot and Werning (2018) tax technology, via a constrained set of taxes, in a complete market with a continuum of agents. The authors find that when it is optimal to tax new technologies, such taxes will likely to be small. Our framework is different. In particular, it doesn't admit contracting between current and future generations. Still, as in Sachs, Benzell and Lagarda (2015), tax and transfer policies could be added to our model to maintain the capital stock and preserve the intergenerational distribution of welfare in the face of AI innovation.

3 The Model

Agents consume two products – corn and prayers.[8] Corn, which can be consumed or invested, is produced using capital and code. The CES function governing corn production can be viewed as a smart machine or robot since it combines physical capital and code. Prayers are produced via a CES function of low- and high-tech labor.[9] Prayers are ephemeral, i.e., consumed when produced. Supplies of high- and low-tech workers remain fixed through time.[10]

3.1 Production

Time-t production of corn, Y_t, and prayers, S_t, satisfy (1) and (2),

$$Y_t = D_Y[\alpha(K_t)^{\frac{\varepsilon_y-1}{\varepsilon_y}} + (1-\alpha)(A_t)^{\frac{\varepsilon_y-1}{\varepsilon_y}}]^{\frac{\varepsilon_y}{\varepsilon_y-1}}, \tag{1}$$

$$S_t = D_S[\gamma(H_{S,t})^{\frac{\varepsilon_s-1}{\varepsilon_s}} + (1-\gamma)(G_t)^{\frac{\varepsilon_s-1}{\varepsilon_s}}]^{\frac{\varepsilon_s}{\varepsilon_s-1}}, \tag{2}$$

where $H_{S,t}$ is the number of high-tech workers in the prayer sector and G_t references low-tech workers. D_S and D_Y are total factor productivity terms, γ and α are CES share parameters, and ε_y and ε_s are CES elasticities. The stock of code A_t grows according to,

$$A_t = \delta A_{t-1} + zH_{A,t}, \tag{3}$$

[8]Frey and Osborne (2013) identify the priesthood, psychotherapy, and coaching as among the occupations least subject to automation.

[9]Adding labor to the production of corn, or capital to the production of prayer, would not alter our qualitative findings.

[10]Our model can readily accommodate long-run balanced growth arising from population growth or labor-augmenting technological change.

where the "depreciation" factor is $\delta \in [0,1)$. Higher δ means that legacy code is useful for longer.[11] $H_{A,t}$ stands for the supply of high-tech coders and z is their productivity coefficient.[12]

The demands for code, high-tech workers, and capital satisfy

$$\max_{K_t, A_t} Y_t(A_t, K_t) - m_t A_t - r_t K_t, \tag{4}$$

where corn is the numeraire, m_t is the rental rate for code, and r_t is the interest rate. Factor demands for prayer reflect,

$$\max_{H_{S,t}, G_t} q_t S_t(H_{S,t}, G_t) - w_t^G G_t - w_t^H H_{S,t}, \tag{5}$$

where q_t is the price of prayer, w_t^H is a high-tech worker's wage in the prayer sector, and w_t^G is a low-tech worker's wage.

Factor prices satisfy

$$w_t^H = q_t D_S [\gamma (H_{S,t})^{\frac{\varepsilon_s - 1}{\varepsilon_s}} + (1-\gamma)(G_t)^{\frac{\varepsilon_s - 1}{\varepsilon_s}}]^{\frac{1}{\varepsilon_s - 1}} [\gamma (H_{S,t})^{-\frac{1}{\varepsilon_s}}], \tag{6}$$

$$w_t^G = q_t D_S [\gamma (H_{S,t})^{\frac{\varepsilon_s - 1}{\varepsilon_s}} + (1-\gamma)(G_t)^{\frac{\varepsilon_s - 1}{\varepsilon_s}}]^{\frac{1}{\varepsilon_s - 1}} [(1-\gamma)(G_t)^{-\frac{1}{\varepsilon_s}}], \tag{7}$$

$$r_t = D_Y [\alpha (K_t)^{\frac{\varepsilon_y - 1}{\varepsilon_y}} + (1-\alpha)(A_t)^{\frac{\varepsilon_y - 1}{\varepsilon_y}}]^{\frac{1}{\varepsilon_y - 1}} [\alpha (K_t)^{-\frac{1}{\varepsilon_y}}], \tag{8}$$

and

$$m_t = D_Y [\alpha (K_t)^{\frac{\varepsilon_y - 1}{\varepsilon_y}} + (1-\alpha)(A_t)^{\frac{\varepsilon_y - 1}{\varepsilon_y}}]^{\frac{1}{\varepsilon_y - 1}} [(1-\alpha)(A_t)^{-\frac{1}{\varepsilon_y}}]. \tag{9}$$

[11] Some of our simulations assume a depreciation rate of 30 percent per period, where a period is roughly 30 to 40 years. This corresponds to a typical company needing to replace approximately 1 percent of its code base annually to maintain the same level of output. The actual rate of code depreciation in the economy is unclear. The IRS allows for a 3 year useful lifespan for licensed software. For software developed in house or purchased bespoke software, costs can be amortized over a 15 year period (as a Section 197 intangible). Software that is bundled with hardware is implicitly assumed to deprecate at the rate of the hardware. On the other hand, many programs created over 50 years ago are still in use, such as those written for older nuclear reactors.

[12] The corn production process can be understood by analogy to a firm whose service is making good chess moves. The firm can improve its service either by increasing the quality of its chess program (increasing its efficiency units of code) or using more capital (computer cores) to exploring winning moves.

3.2 Households

Whether high- or low-tech, households maximize

$$u = (1-\phi)[(1-\kappa)logc_{y,t} + \kappa logs_{y,t}] + \phi[(1-\kappa)logc_{o,t+1} + \kappa logs_{o,t+1}], \tag{10}$$

subject to,

$$c_{y,t} + q_t s_{y,t} + \frac{c_{o,t+1} + q_{t+1}s_{o,t+1}}{1+r_{t+1}} = i_{j,t}, \tag{11}$$

where $c_{y,t}$, $c_{o,t}$, $s_{y,t}$, $s_{o,t}$, are consumption of corn and prayer by the young and old, respectively, and $i_{j,t}$ is total resources of group j. For low-tech workers,

$$i_{G,t} = w_t^G. \tag{12}$$

For high-tech workers in the prayer sector,

$$i_{(H,S),t} = w_t^H, \tag{13}$$

and for high-tech workers writing code,

$$i_{(H,A),t} = z(m_t + \delta p_t), \tag{14}$$

where zm_t is revenue from renting out newly produced code and $z\delta p_t$ is the proceeds from the sale of rights to future use of this fresh code.

Households save in the form of capital and code. Capital and code accumulation obeys

$$\phi I_t = K_{t+1} + p_t \delta A_t, \tag{15}$$

where I_t is the total resources of those born in t, ϕ is the saving propensity of the young, and $p_t\delta A_t$ is the value of code retained from the current period. In equilibrium the return to both investments is identical.

Figure 1 summarizes the timing of consumption and saving decisions for high-tech workers in the corn industry.

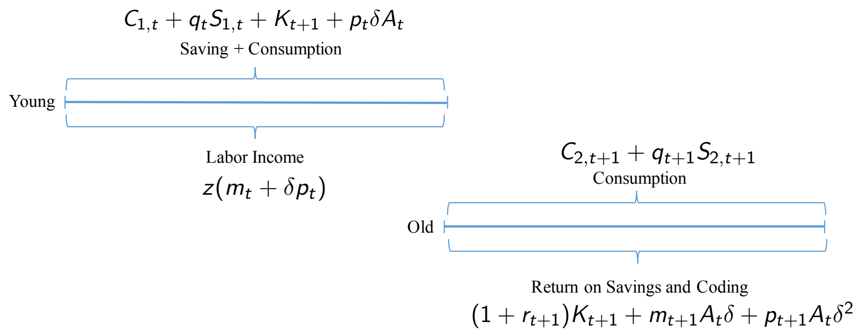


Figure 1: Summarizing a high-tech worker's flow budget constraints$t$.

Demands satisfy,

$$s_{y,t} = \frac{\kappa(1-\phi)i_{j,t}}{q_t}, \tag{16}$$

$$c_{y,t} = (1-\kappa)(1-\phi)i_{j,t}, \tag{17}$$

$$s_{o,t+1} = \frac{1 + r_{t+1}}{q_{t+1}}[\kappa\phi i_{j,t}], \tag{18}$$

and

$$c_{o,t+1} = [1 + r_{t+1}][(1-\kappa)\phi i_{j,t}]. \tag{19}$$

3.3 Equilibrium

Since high-tech workers are mobile between sectors,

$$w_t^H = z(m_t + \delta p_t). \tag{20}$$

Asset-market clearing entails equal returns on capital and code, i.e.,

$$p_t = \sum_{s=t}^{\infty} R_{s+1,t}^{-1} \delta^{s-t} m_{s+1}, \tag{21}$$

where $R_{s,t}$ is the compound interest factor between t and s, i.e.,

$$R_{s,t} = \prod_{j=t}^{s}(1 + r_j). \tag{22}$$

Finally, equilibrium requires

$$Y_t = C_{y,t} + C_{o,t} + K_{t+1} - K_t, \tag{23}$$

$$H_t = H_{A,t} + H_{S,t}, \tag{24}$$

and

$$S_t = S_{y,t} + S_{o,t}, \tag{25}$$

where C_y, C_o, S_y, S_o, are total consumption demand of corn and payer by the young and old respectively.

3.4 The Steady State with Cobb-Douglas Production

If production functions are Cobb-Douglas, the steady state is implicitly defined by the following two equations in $k = \frac{K}{A}$, the capital to code ratio, and q, the relative price of prayer. Derivation details are reported in appendix B.

$$\begin{aligned} D_y k^{\alpha} =& [\frac{(1-\phi)(1-\kappa)}{\phi}][k + \frac{(1-\alpha)D_y k^{\alpha}\delta}{1+\alpha D_y k^{\alpha-1} - \delta}] \\ &+ (1-\kappa)[k + \frac{(1-\alpha)D_y k^{\alpha}\delta}{1+\alpha D_y k^{\alpha-1} - \delta}][1 + \alpha D_y k^{\alpha-1}] \end{aligned} \tag{26}$$

and

$$k + p\delta = \phi[z(m + p\delta)H + (1-\gamma)G(\frac{\gamma}{z(m+p\delta)})^{\frac{\gamma}{1-\gamma}}(qD_s)^{\frac{1}{1-\gamma}}], \tag{27}$$

where,

$$m = (1-\alpha)D_y k^{\alpha}, \tag{28}$$

$$r = \alpha D_y k^{\alpha-1}, \tag{29}$$

$$p = \frac{(1-\alpha)D_y k^{\alpha}}{1 + \alpha D_y k^{\alpha-1} - \delta}. \tag{30}$$

The mechanism for immiseration in the model is low wages. All else equal, a higher capital-to-code ratio spells lower wages. In the Cobb-Douglas case, we have

$$\frac{dk}{d\delta} = -\left(\frac{-\left[-(1-(1-\kappa)\alpha) - \frac{(1-\kappa)(1-\alpha)}{\phi} \right] D_y - \frac{(1-\kappa)}{\phi} k^{1-\alpha} + (1-\kappa)(1-\alpha)\alpha D_y^2 k^{\alpha-1}}{\frac{(1-\alpha)(1-\kappa)(1-\delta)}{\phi} k^{-\alpha} - (1-\alpha)[(1-\kappa)(1-\alpha)\delta\alpha - \alpha(1-(1-\kappa)\alpha)] D_y^2 k^{\alpha-2}} \right) \tag{31}$$

One can readily choose parameter values that make this derivative negative.

3.5 Analytic Results Illustrating Key Mechanisms in a Simplified Model

Our model's dynamics are analytically tractable under an extreme simplified assumption. Consider the special case of only one good (corn) produced using Leontief technology (i.e. $\kappa = 0$, $G = 0$, $\epsilon_y \to 0$).[13] Under the right conditions, a sufficiently large increase in the code retention rate zeros out the wage, producing full economic collapse. This occurs either after two periods or at a specific date in the future. This simplified model exhibits many key features of the full model. These include the potential for, in this case, an abrupt boom and bust.

Assume production satisfies

$$Y_t = D_y min[(1-\alpha)A_t, \alpha K_t]. \tag{32}$$

Since all workers are coders and their period-specific labor supply, H, is fixed, A_t is easy to derive. Assuming δ is zero prior to time zero and fixed at $\hat{\delta}$ afterwards,

$$A_t = \frac{zH(1-\hat{\delta}^{t+1})}{1-\hat{\delta}}. \tag{33}$$

When code is scarce, production is linear in code and the marginal product of code and the wage are positive.

[13] Unlike the main model, we also assume no physical capital depreciation. Investment as a storage technology motivates saving in the absence of a positive marginal product of capital.

The return to capital is zero. The opposite holds when capital is scare. I.e.,

$$m_t = \begin{cases} D_y(1-\alpha) & \text{if } (1-\alpha)A_t < \alpha K_t \\ 0 & \text{if } (1-\alpha)A_t > \alpha K_t \end{cases}$$

and

$$r_t = \begin{cases} 0 & \text{if } (1-\alpha)A_t < \alpha K_t \\ D_y\alpha & \text{if } (1-\alpha)A_t > \alpha K_t. \end{cases}$$

Economic collapse, if it occurs, arises in the period after code switches from scarce to abundant. Let T reference the first date at which code becomes abundant, assuming this occurs. Then $m_t = D_y(1-\alpha)\ \forall\ t < T$ and $m_t = 0\ \forall\ t \geq T$. The economy permanently collapses at $T+1$.

The condition for code abundance is $\alpha K_T < (1-\alpha)A_T$. Inserting equations (13) and (14) and rearranging yields

$$\alpha(\phi H z m_{t-1} + p_{T-1}\hat{\delta}(\phi z H - A_{T-1})) < (1-\alpha)A_T. \tag{34}$$

Note, from equation (33) that A_{T-1} is greater than or equal to zH. Hence, the second term on the left-hand-side of equation (34) is negative. Because only a fraction ϕ of newly produced code's value is saved, a positive price of code serves, on balance, to crowd out capital. When ϕ, the saving preference parameter, is large, immiseration is less likely.

Also note that $m_T = 0$ implies $p_{T-1} = 0$ since code is valueless if it is expected to have no return in subsequent periods. Setting $p_{T-1} = 0$ and incorporating formulas for m_t and A_t yields a condition for the economy to collapse at time $T+1$ as a function of parameters

$$\alpha\phi D_y < \frac{1-\hat{\delta}^{T+1}}{1-\hat{\delta}}, \tag{35}$$

As equation (35) indicates, if collapse occurs, it occurs faster the smaller the product of ϕ, α, and D_y, and the larger the value of $\hat{\delta}$. Output rises until the economy collapses since A_t is strictly increasing over time.

Neither the timing nor occurrence of economic collapse is affected by coder productivity, z.

What happens to welfare? Those born at $t = 0$ through $t = T - 2$ are better off than those born prior to period 0. The reason is their ability to sell their durable code raises their total wage. Those born in $T - 1$ are also better off. They receive the $t = 0$ wage while young, but a high interest rate on their physical capital investments while old. As for generations born in $t = T$ or later, they starve.

Equation (35) also specifies the condition for permanent scarcity of code, namely,

$$1 - \frac{1}{\alpha\phi D_y} > \hat{\delta} \tag{36}$$

If code is permanently scarce, then $w_{H,t} = zm_t + \frac{zm_t}{1-\hat{\delta}} = z(1-\alpha)D_y + \frac{z(1-\alpha)D_y}{1-\hat{\delta}}$. Hence, so long as $\hat{\delta}$ is sufficiently small (or D_y, ϕ, and α are sufficiently large), a rise in δ leads to an immediate and permanent increase in compensation and welfare.[14] The size of the increase in wages is increasing in δ, z and D_y.

This simplified model also illustrates the role of the elasticity of substitution in the corn sector on the possibility of immiseration. If code and capital were perfect substitutes rather than complements, an increase in δ from zero to positive would never lower the marginal product of code, capital, or the wage. Therefore, strong complementarity between code and capital makes imisseration more likely.

4 Numerical Solution and Simulation Results

The models' main novelty is the inclusion of the stock of code in the production of goods. We calculate the economy's perfect foresight transition path following an immediate and permanent increase in the rate of code retention (decline in the code depreciation rate). The solution is via Gauss-Seidel iteration.

[14]Compensation of coders jumps immediately to a higher level where it remains indefinitely if the economy never collapses (if the economy does collapse in the future, the present discounted value of code falls as collapse nears). Interestingly, although new coders get paid the same through time and capital income is zero, output rises, albeit at a decelerating rate. In the steady state, total wage income is equal to total output. Along the transition, the high wage is financed by decummulation of physical capital, which is increasingly crowded out.

4.1 Simulation Results

When the code retention rate, δ equals zero, corn production is conventional – based on contemporaneous inputs of capital and labor (code writers). But when δ rises, corn production depends not just on capital and current labor, but also, implicitly, on deceased high-tech workers.

The increase in δ initially raises the compensation of coders. This draws more high-tech workers into coding, raising high-tech worker compensation in both sectors. For most parameterizations, the concomitant reduction in the supply of prayers raises the price of prayers. Depending on the degree to which high-tech workers complement low-tech workers in producing prayer, the wages of low-tech workers will rise or fall. When the two forms of labor are close substitutes the wages of low-tech workers track those of the high tech. When low-tech workers require high-tech workers to complement them, their wages fall as high-tech workers depart for the corn industry.

The situation of high-tech workers degrades over time. As more durable code comes on line, the marginal productivity of code falls, making new coders increasingly redundant. Eventually the demand for coders is limited to those needed to cover the depreciation of legacy code, i.e., to retain, retrain, maintain, and update the AI. The remaining high-tech workers find themselves working in the prayer sector. The upshot is that high-tech workers can end up potentially earning far less than in the initial steady state.

What about low-tech workers? The price of prayers peaks and then declines thanks to the return of high-tech workers to the sector. This puts downward pressure on low-tech workers' wages and, depending on the complementarity of the two inputs in producing prayer, low-tech workers may also see their wages fall. When low-tech workers are close substitutes for high-tech workers, the boom-bust in high-tech workers' compensation generates a boom-bust in low-tech compensation. In the special case where high and low-tech workers are perfect substitutes, their wages move in lock step.

The economy's dynamic reaction to a higher δ operates in part through the impact on capital formation. The initial rise in earnings of at least the high-tech workers can engender more aggregate saving and investment. The increased capital makes code and, thus, high-tech workers more productive. But if the total compensation of workers eventually falls, so too will the saving of the young and the economy's supply of capital. Less capital lowers the marginal productivity of code and raises interest rates. This lowers the price of code and the wages of those who produce it.

We next consider alternative transition paths arising under different technology or preference assumptions.

We first consider paths that feature immiserating growth. Next we show that the opposite is possible - long-run, welfare improving growth. We also consider a range of other possibilities, including those that involve a changes in the relative wage positions of high- and low-tech workers. Each simulation features an immediate and permanent rise in the code-retention rate. But the dynamic impact depends on the size of the shock and the choice of parameters.

4.2 Illustrating Long-Run Immiseration

Table 1 reports model parameters for our first simulation. In this section, we assume that workers save 20 percent of their wages. This may seem very low, but it entails an initial steady-state marginal product of capital of 1.66. Taking a period as 30 years, this is 1.02 percent on an annual basis. This is far below empirical estimates of the U.S. economy's marginal product of capital (pre-tax, but post-depreciation).[15]. Thus saving is arguably calibrated to be over abundant in this illustration.

Parameters for Immiserating Growth

Parameter Description	Model Parameter	Value
Elasticity in Prayer Sector	ε_s	∞
Elasticity in Corn Sector	ε_y	1
Prayer High-Tech Input Share Param.	γ	0.5
Corn Capital Input Share Param.	α	0.5
Code Retention Rate	δ	0 shocked to 0.7
Saving Preference Param.	ϕ	0.2
High-Tech Worker Quantity	H	1
Low-Tech Worker Quantity	G	1
Prayer Consumption Share	κ	0.5
Code Writing Productivity	z	1
TFP in Corn	D_y	1
TFP in Prayer	D_s	1

Table 1: This table gives parameter values for the first illustration (i.e. Illustrating Long-Run Immiseration) of the effects of a permanent increase in the retention rate, δ, from zero to .7. We take the intermediate value of .5 for κ, α, and γ. The productivity terms z, D_Y, and D_S, are set to one. The second illustration (i.e. Illustrating Long-Run Welfare Improvement) assumes identical parameters except $\phi = .85$.

Figure 2 shows that technological growth, namely the code-retention rate, δ, rising from 0 to .7, can have negative long-term consequences. As the top left panel indicates, real national income rises several percent, peaking at 7.8 percent above baseline a period after the shock.[16] But it ultimately declines, ending up 4.2

[15] See, for example, (Poterba 1998)

[16] Unless otherwise noted, national income, personal income, and wages are reported in real terms. The price index used is a geometric mean of the relative price of corn and prayer. This is the correct price index to use when utility is Cobb-Douglas. The weights used are their corresponding shares in consumption. The price index is

$$\Pi_t = q_t^{\frac{q_t(S_{y,t}+S_{o,t})}{q_t(S_{y,t}+S_{o,t})+C_{y,t}+C_{o,t}}} = q_t^{\kappa} \tag{37}$$

Output of corn and prayer are reported in their own units. Other prices (the price of code, p, interest rate, r, and price of prayer, q) are reported in units of the numeraire (corn) unless otherwise noted. National income is the sum of all payments to

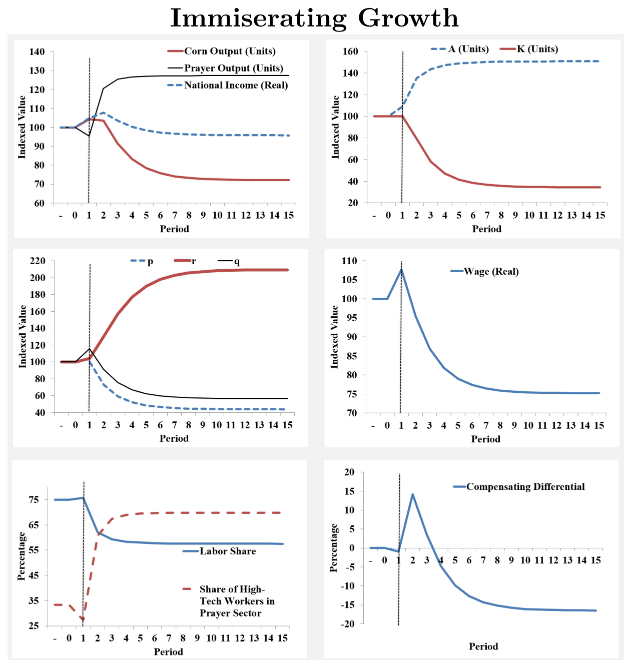


Figure 2: Transition paths based on table 1. In this and all subsequent figures, compensating differential references the percentage change in steady-state consumption needed for initial steady-state households' lifetime utility to equal the lifetime utility of households who are old in a given period along the transition path. Output, capital and code stocks are in units. Wage and national income are deflated.

percent below its initial steady-state value. In the long-run, corn output decreases 28.0 percent.

The output of prayer dips after the shock, as workers migrate to lucrative coding jobs, then increases steeply as the automated workers return. The price of prayers does the opposite. While production of prayers is 27.4 percent higher in the long run, prayers, the long-run price of prayers is 43.4 percent lower than before the technological breakthrough. Both types of workers are perfect substitutes in spirituality, which, in our simulations, always employ both. Their common compensation initially jumps 7.6 percent and then falls gradually. In the long run all workers earn 24.8 percent less than was originally the case.

What happens to the welfare of different agents through time? This is shown in the bottom right corner of figure 2.

The first generation potentially impacted is the one that is young in the period before the code retention shock and old during the period of the shock. They own zero corn, hence, they are only impacted through price effects. This generation benefits from a higher marginal product of capital, but are harmed by a higher price of prayer. On net, this generation is slightly worse off.

workers and capital.

The next generation is the first to benefit from the code retention shock. Members of this generation know that they will gain an additional benefit from working in the corn sector – the value of their stored code. This is the generation that provides the largest share of its labor to the corn sector, and this generation's total compensation when young is high. The marginal product of capital one period after the code retention shock is also high because of the relative abundance of code, which is a complement to capital. This generation, which benefits both from high total compensation when young and a high marginal product of capital when old, is, however, forced to pay a higher price of prayer. On balance, they experience a 14.2 percent rise in lifetime utility, measured as a compensating differential, relative to generations in the initial steady-state. Generations in the long run are worse off relative even to those alive in the pre-shock steady state. Relative to the latter, those born in the long run are 16.5 percent worse off and national income is 4.2 percent lower.

The top right chart helps explain why good times presage bad times. The stock of code shoots up and stays high. But the stock of capital immediately starts falling. After six periods there is over 50 percent more code, but 65 percent less capital. The marginal product of capital skyrockets, increasing the long-run interest rate 110.2 percent above its initial steady-state value.

The huge long-run decline in the capital stock and associated rise in its marginal product has two causes. First, as just stated, wages, which finance the acquisition of capital, fall almost in half by the implicit competition with deceased workers. Second, the advent of a new asset – durable code – crowds out capital accumulation. When δ rises, all workers immediately enjoy an increase in their compensation. This leads to more saving, but not necessarily in the form of capital. Instead, much of the short-lived extra saving is used to acquire claims to legacy code. Initially, when the stock of code is small, its price is very high. Later, when the stock of code is large, its price is quite low. But the product of code's price and its quantity are always sufficiently high to crowd out investment in capital.

What happens to labor's share of national income? Initially, it rises slightly from 75.0 percent in the initial steady state to 75.7 percent. It then declines, falling to 57.5 percent in the long run. This reflects the higher share of output paid to legacy code. This long-run decline in both physical capital and labor's shares of national income arise in all our simulations.

4.3 Illustrating Long-Run Welfare Improvement

Figure 3 shows that the tech boom need not auger long-term economic decline. A higher saving propensity is the key. In the immiserating growth case, we assumed a saving propensity, ϕ, equal to .2. Here, keeping

Welfare Improving Growth

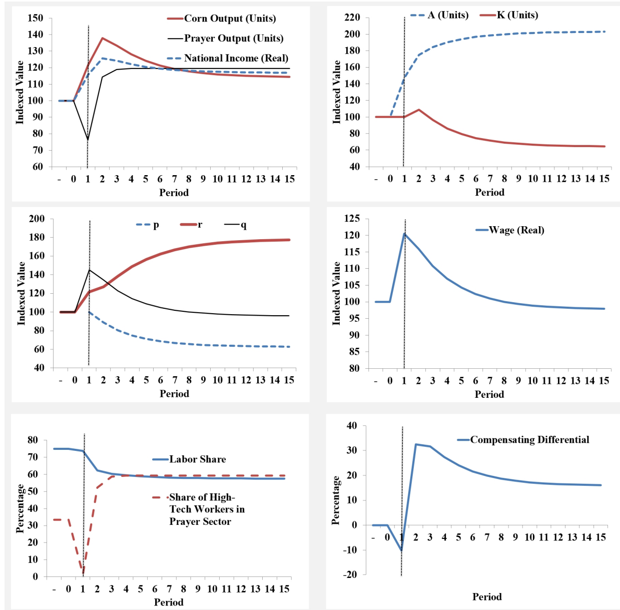


Figure 3: Transition paths based on table 1, with the exception of a higher saving rate ($\phi = .85$).

all other parameters fixed, we assume ϕ equals .85. With this far higher saving rate, workers are better off in the long run. However, in the first period, when the code-retention shock hits, the generation that is already old is worse off due to the short-term rise in the price of prayer.

National income peaks in the period after the technological shock, rising 25.6 percent above baseline. But in the long-run, national income is only 16.8 percent higher. This reflects a rise and then fall in the capital stock, but not one that is sufficient to reduce long-run welfare. In the prior simulation the capital stock immediately declined. Here the capital stock temporarily increases 8.7 percent above its initial value.

A short-run rise in both the capital stock and prayer price boosts the common wage in the short run and leaves it at roughly its initial value in the long run. After peaking 20.5 percent above its initial value, the wage falls, ending up 2.2 percent lower. The stock of code ends up more than doubling. But the capital stock, notwithstanding the high rate of saving, ultimately declines by 35.8 percent.

The respective increase and decrease in the stocks of code and capital produce a significant rise in the economy's interest rate – 78.0 percent in the long run. Although the common wage of high and low-tech workers decreases slightly from its initial level, the rise in the interest rate permits future generations to consume significantly more.

Why does a high enough saving rate lead the δ shock to increase long-run welfare? The answer is that whatever happens to the stock of code, a higher saving rate entails a higher capital stock and, therefore, less of a decline in wages after code accumulates.

At the end of this section we explore the sensitivity of welfare changes to the combination of the saving propensity and code retention rates. The combination of low saving and high code retention rates lead to the most negative outcomes. Conversely, a high saving preference and high code retention rate lead to the best outcomes.

4.4 The Wide Range of Possible Outcomes

As just demonstrated, the model's reaction to the δ shock is highly sensitive to parameter values. Figure 4 focuses on the dependence of the outcome on the saving preference parameter and code writing productivity. It presents two contour maps of the long-run compensating differential. It's top half considers combinations of saving preference parameters ϕ and shocks to δ assuming table 1's values of the other parameters. Because the two types of workers are perfect substitutes, the compensating differential for both types is the same. Redder areas reference higher long-run utilities relative to the initial steady state. Bluer areas reference the opposite. Long-run utility increases most when δ is large and the saving rate is high. It decreases the most when the δ shock is high and the saving rate is low.

Figure 4's lower half considers joint shocks to the saving rate and code-writing productivity, z. Higher values of each reinforces their individual positive impacts on long-run utility. As opposed to δ shocks, increases in code-writing productivity (z) always enhance all agents' welfare. The reason is simple – this shock makes living, but not deceased high-tech workers more productive. Increasing labor's productivity in other tasks has the same result. As this model posits no disutility from labor, reducing labor's productivity is isomorphic to restricting its supply. Policies that attempt to raise wages by reducing labor supply - such as increasing the minimum wage - will, therefore, backfire.

Appendix table 2 shows additional findings for other parameter combinations. The table's baseline simulation (row one) assumes intermediate parameter values. Subsequent rows show the impact of sequentially modifying one parameter. A very large range of outcomes is possible. One interesting dynamic possibility is 'inequality flipping'. In these parameterizations, an increase in δ increases the wage of high-tech workers above that of low-tech workers for a single generation, but as code accumulates, it is low-tech workers who disporportionately benefit in the long run.[17]

[17] Full simulated endogenous variable paths for an example available upon request.

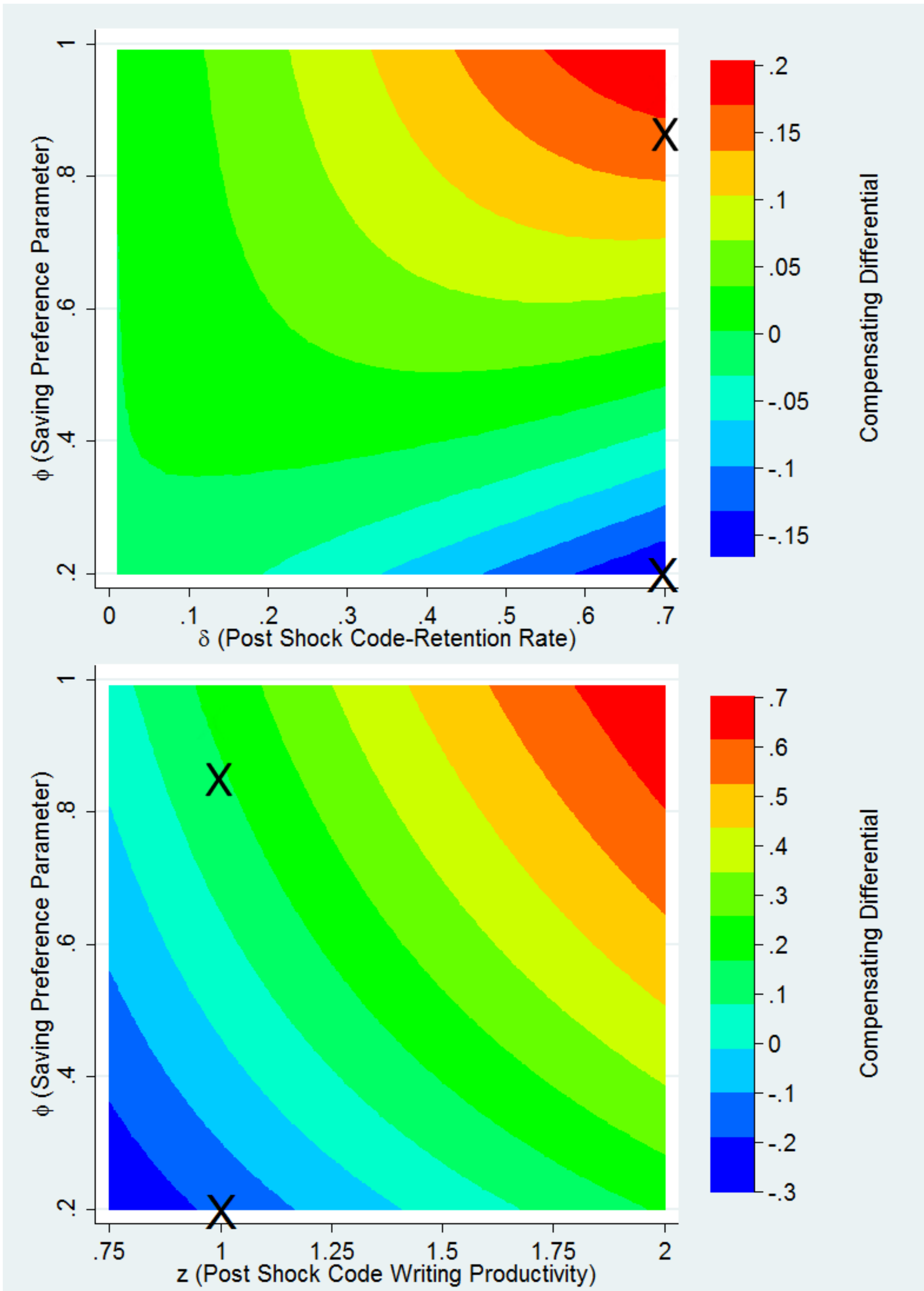


Figure 4: Long-Run Compensating Differential for Alternative Saving and Code-Retention and Productivity Shocks. Parameters not on axes are given in table 1. X's denote parameter combinations with transition paths discussed in the text.

The range of simulation results reported in appendix table 2 teach several new things. First, high-tech workers benefit from higher substitutability between capital and code in the corn sector. This makes sense. Indeed, when capital and code are perfect substitutes, corn production is linear in the sum of the two inputs. Hence, the marginal product of code is unchanged in response to a rise in the stock of code. Consequently, compensation to coders is unaffected by a rise in code retention and there is no mechanism for a fall in workers' saving and investment or for long-run immiseration.

Second, with both Cobb-Douglas production and preferences, the path of the capital-to-code ratio in response to a rise in delta, starting from $\delta = 0$, is independent of the absolute and relative numbers of each type of worker.[18]

Third, an increase in δ always produces a tech boom with increases in both the price of code and the wage of high-tech workers.[19] In most simulations, the boom is short lived, auguring a major tech and saving bust. Finally, the δ shock generally raises labor's share in the short run and lowers it in the long run.

5 The Role of Property Rights and Rivalry

To this point we've assumed that code is private and rival. Specifically, we've assumed that when one firm uses code it is unavailable for rent or use by others. But unlike physical capital, code represents stored information, which may be non-rival in its use. Non-rivalry does not however necessarily imply non-excludability. Patents, copyrights, trade secrets, and other means can be used to limit code's unlicensed distribution. On the other hand, the government can turn code into a public good by mandating it be open source. There are significant open questions about the legality and desirability of IP protection for AI and

[18] Consider a doubling of H. This will double H_Y in the $\delta = 0$ economy. But if H_Y also doubles along the entire transition path, the path of k will remain unchanged. One can see this by combining the equation for market-clearing in capital with that for market-clearing in code. This, of course, requires that the path of H_S be twice as large as well. But this outcome as well as a doubling of q_t is implied by (16). This k-path invariance to initial levels of H and G is somewhat surprising and suggests that transforming more low-tech into high-tech workers may have less impact on the economy than one might have thought. Still, such a policy, if enacted before the rise in delta, would lower the real wages of skilled workers. (Their wages valued in corn wouldn't change, but the higher price of prayer would lower their real wage.) It would also improve the relative welfare of those who remain unskilled workers since their wage measured in units of capital will rise thanks to the higher marginal revenue of their labor. Additional effects would arise were H or G to vary once delta had risen and the economy was in transition. In this case, the k path would temporarily fall making code and coding less valuable. However, in the long run, the real wages of each type of worker are independent of such transition effects on the path of k.

[19] An increase in δ from zero to a positive value must create an initial increase in the high-tech wage. This is because the capital stock is fixed, and if high-tech workers maintain their initial occupational mix, the initial marginal product of code would be fixed. If the marginal product of code is fixed, the total wage will increase (because the code produced goes from a zero to a positive future rental price). Some high-tech workers may move from the prayer sector to making code for the corn sector, but this can only occur if the wage still increases on net (high-tech workers moving to the corn sector raises the price of prayer and the marginal product of high-tech labor in that sector).

original works created by AI systems (Center for the Fourth Industrial Revolution 2018).

This section modifies the baseline model to investigate these questions. To do so we add a firm entry decision. Firm entry is important in the context of IP because when code can be simultaneously used by multiple parties, the number of these parties must be determined. Corn producing firms enter by paying a fixed cost each period, and gain access to the amount of free code available. This fixed cost corresponds to both the overhead necessary to run a business and the cost of discovering a new idea for applying AI. When code is excludable, firms may also rent an additional supply of it at a market price. Strong IP protections for AI incentives the creation of more code but limit the use of AI that already exists.[20]

We first present the modified model in the baseline case of private (rival and excludable) code. We then explore two alternative scenarios. In the first, code is non-rival and non-excludable, i.e., it is a public good. In the second, code is non-rival, but excludable. In other words, those who develop AI can rent it out to as many companies as they like without friction.

5.1 Rival, Excludable (Private) Code

Corn firms maximize their profit, which is equal to

$$\pi_{j,t} = F(k_{j,t}, zH_{j,t} + a_{j,t} + \overline{A}) - C - r_t k_{j,t} - m_t a_{j,t} - w^H H_{j,t}, \tag{38}$$

where $\pi_{j,t}$ are profits for firm j at time t, $F(\bullet)$ is a constant elasticity of substitution production function, $k_{j,t}$ is the amount of capital rented by the firm, $a_{j,t}$ is the amount of code rented by the firm, $H_{j,t}$ is the amount of high-tech labor hired by the firm, $\overline{A}$ is the exogenously set amount of free code in the economy, and C is the per-period fixed operating cost. In equilibrium all firms have zero profits.

Market clearing conditions are,

$$\sum a_{j,t} = \delta A_{t-1}, \tag{39}$$

$$\sum k_{j,t} = K_t, \tag{40}$$

$$\sum H_{j,t} = H_{A,t}, \tag{41}$$

$$Y = c_{o,t} + c_{y,t} - K_t + K_{t+1} - NC, \tag{42}$$

[20]We incorporate some amount of free public code in all institutional scenarios to ensure entry.

where N is the number of firms. All other equations are as in the baseline model. Since all firms are identical, (42) can be converted into an equation for N, the number of firms.

$$0 = NF(\frac{K_t}{N}, \frac{zH_{A,t}}{N} + \frac{1}{N}\delta A_{t-1} + \overline{A}) - NC - r_t K_t - m_t \delta A_{t-1} \tag{43}$$

Firms enter up to the point that the value of the public code they obtain for free, namely $\overline{A}$, equals their fixed cost of production. Thus,

$$\overline{A} F_{a,t} = C. \tag{44}$$

This fixes the marginal product of code at $\frac{C}{\overline{A}}$ in every period. Intuitively, new firms can acquire a perfect substitute for new code, and, thus, new coders at a fixed cost by setting up shop and gaining access to $\overline{A}$ in free code. Given that corn's production obeys constant returns to scale, fixing code's marginal product means fixing the ratio of capital to code. This, in turn, fixes the interest rate. Hence, the rental rates of code and capital are invariant to the increase in δ.

To solve the model an additional step is added to the iteration procedure. Given a guess of prices and stocks in a period, (43) is used to calculate N. This guess of N in each period is included in the next iteration to calculate new prices.

Figure 5 shows transition paths for this economy, with excludable, non-rival code, after an increase in the code retention rate. Parameter values are provided in table 2. Although an increase in δ does not change the marginal productivity of code, it does raise coder compensation. The reason is that coders can now sell property rights to the future use of their invention. Immediately after the shock, the wage of high-tech workers increases 10.2 percent, decreasing to 6.5 percent higher in the long run due to a decrease in the relative price of corn.

The number of firms in the economy decreases as a result of the technology shock, by 11.3 percent in the long run. Were the number of firms to remain fixed, the jump in δ would entail a higher code to capital ratio (in the short term the capital stock is fixed, and in the long run it increases by less than the code stock). This would mean a lower marginal productivity of code, which equation (44) precludes. It would also mean a negative payoff to setting up a new firm. Another way of viewing this relationship is that as the rental price of code decreases, the attractiveness of acquiring code by setting up a new firm decreases. As the latter is fixed, the rental price of code is fixed, with the margin of adjustment being fewer firms created.

Parameters for Institutional Simulations

Model Parameter	Role	Value
ε_s	Elasticity in Prayer Sector	1
ε_y	Elasticity in Corn Sector	1
γ	Prayer High-Tech Input Share Param.	0.5
α	Corn Capital Input Share Param.	0.5
δ	Code Retention Rate	0 shocked to 0.25
ϕ	Saving Rate	0.5
H	High-Tech Worker Quantity	1
G	Low-Tech Worker Quantity	1
κ	Prayer Consumption Share	0.5
z	Code Writing Productivity	1
D_y	TFP in Corn Sector	1
D_s	TFP in Prayer Sector	1
C	Firm Setup cost	.055
$\overline{A}$	Exogenous Free Code	.25

Table 2: This table gives parameter values for illustrations of the effects of a one-time, permanent increase in the retention rate, δ, from zero to .25 given different institutional settings.

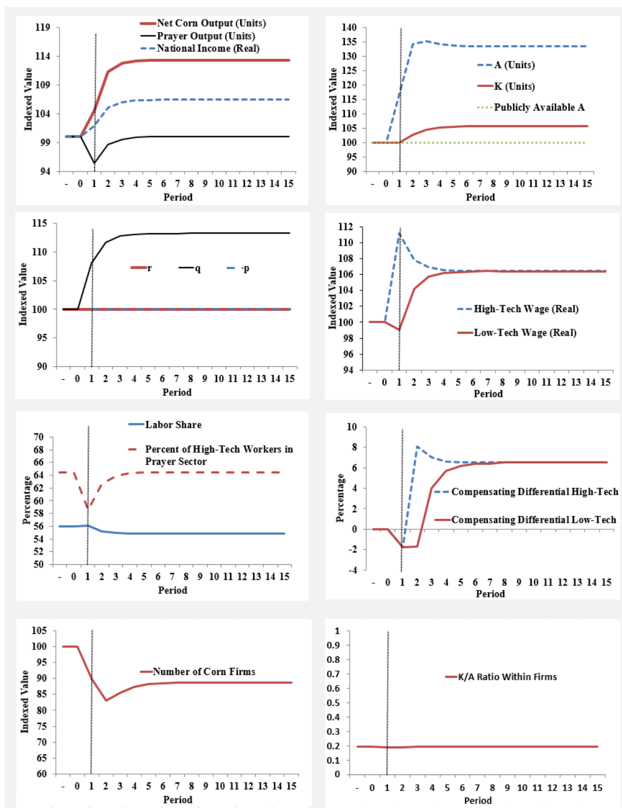


Figure 5: Transition paths based on table 2's parameters and rival, excludable (Private) Code.

In the period of the shock, welfare for the old decreases by 1.8 percent. This is because the cost of prayer increases 8.0 percent while the interest rate remains fixed. In the long run, high and low-tech workers are 6.5 percent better off. As the wage of high-tech workers can only increase as a function of the technology shock, this model variant does not admit long-run immiserating growth absent additional assumptions. For example, if the number of firms were to be fixed due to oligopilization of the industry, or if the fixed cost of firm entry were increasing in the number of firms, (44) would not hold, in which case the marginal productivity of code would decrease as code accumulates. This would reintroduce the possibility of immiserating growth.

5.2 Non-Rival, Non-Excludable (Public) Code

Consider next the case that code, in the period after it is produced, becomes a pure public good used simultaneously by every firm. This could arise by government edict, the wholesale pirating of code, or reverse engineering.

Profits are now

$$\pi_{j,t} = F(k_{j,t}, zH_{j,t} + a_{j,t} + \overline{A}) - C - r_t k_{j,t} - w^H H_{j,t}, \tag{45}$$

as firms no longer need to rent their stock of code ($a_{j,t}$), where

$$a_{j,t} = \delta A_{t-1} \forall j \tag{46}$$

As before, firm entry and exit imply zero profits,

$$0 = NF(\frac{K_t}{N}, zH_{A,t} + \delta A_{t-1} + \overline{A}) - NC - r_t K_t - w^H H_{A,t}. \tag{47}$$

and, because the amount of free code available to newly set up firms changes over time, (44) is modified to

$$(\delta A_{t-1} + \overline{A}) F_{a,t} = C. \tag{48}$$

Finally, with investment in code no longer crowding out investment in capital,

$$K_{t+1} = \phi I_t. \tag{49}$$

Figure 6 shows the transition path after a δ increase for the case of non-excludable code. The initial steady state is the same as in the prior case of excludable rival code. However, the response to the jump in δ is dramatically different. It has no immediate effect on the high-tech wage because workers no longer hold copyright to their code. They, therefore, have no incentive to move to the corn sector, leaving the economy unresponsive to the shock in the short term.

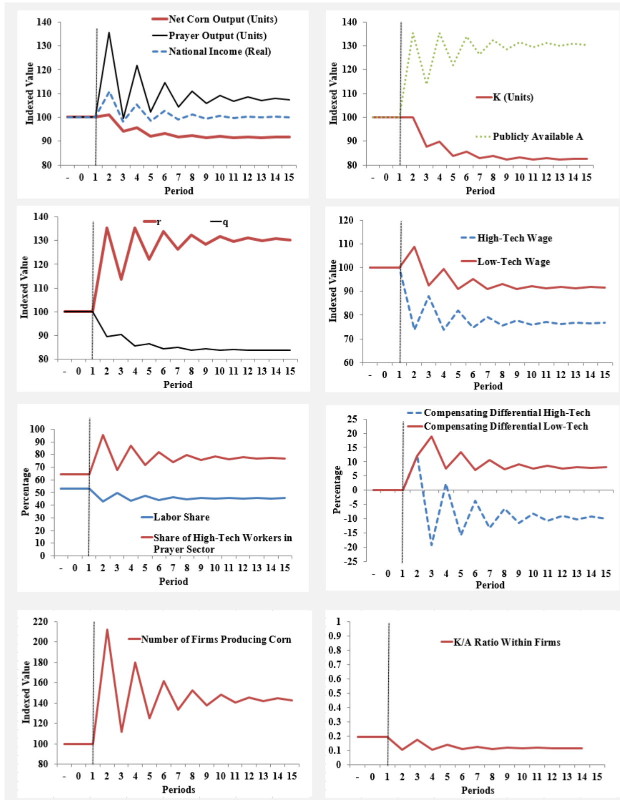


Figure 6: Transition paths based on table 2's parameters and non-rival, non-excludable (Public) Code

In the period after the shock, the economy begins to react. The stock of free public code, which now includes both $\overline{A}$ plus all of the economy's legacy code, is larger. This induces more firm entry. The number of firms immediately more than doubles to 112.4 percent above its initial level in the short run and is 43.4 percent higher in the long run. As indicated in (48), with more free code available, the break even condition entails a lower rental rate of code. In equilibrium, this entails more firms operating with less capital per unit of code.

The lower marginal product of code and, thus, of coders leads 30.3 percent of high-tech workers to move from coding into prayer in the period after the shock. National income peaks at 10.8 percent above its initial level in this period. The interest rate rises by 35.5 percent and the wage of low-tech workers increases by 15.0 percent.

The economy's transition is characterized by a series of damped oscillations. Periods of relatively high coder hiring and fewer firms is followed by periods of plentiful free code, more firm entry, and relatively low coder hiring. In the long run, the share of high-tech workers coding is 12.6 percent higher than its initial level and the high-tech wage is 23.4 percent lower. Welfare in the long run for high-tech workers is 9.7 percent lower. For low-tech workers, welfare is 8.0 percent higher. It is easy to select parameters such that both groups are worse off. As in the baseline model, the main mechanism for immiseration is the reduction of the high-tech wage leading to less capital accumulation. A contributing factor is the inefficiency introduced due to coders no longer being able to internalize the full value of their work.

5.3 Non-Rival, Excludable Code

Another possibility is that code is excludable, but non-rival in its use, permitting high-tech workers to license all their code to all firms in the period after it is produced. The equations for the rival, excludable model hold with the following exceptions. First, profits are given by

$$\pi_{j,t} = F(k_{j,t}, zH_{j,t} + \delta A_{t-1} + \overline{A}) - C - r_t k_{j,t} - m_t \delta A_{t-1} - w^H H_{j,t} \tag{50}$$

Second, the price of code reflects its use by all firms.

$$p_t = \sum_{s=t}^{\infty} R_{s+1,t}^{-1} \delta^{s-t} m_{s+1} N_{s+1}. \tag{51}$$

As figure 7 shows, the δ shock produces a long-run, welfare-improving growth path, indeed a significantly better path than in the rival, excludable case. As in the rival, excludable case, firm entry satisfies equation (44). Hence, the interest rate and marginal product of new code are fixed, the wage of high-tech workers must increase, and a long-term welfare improvement is ensured. Welfare for the old in the period of the shock decreases by 2.6 percent, slightly more than in the rival, excludable case. Thereafter, however, households are much better off, enjoying 11.5 percent higher utility in the long run, measured, as always, as a compensating differential.

This case features less entry. In the long-run, there are 20.8 percent fewer firms than before the δ shock. In the public code case, there is a 43.4 percent increase in the number of firms in response to the shock. In the private code case, the shock reduces the number of firms by 11.3 percent. Intuitively, since each firm can use all available code, fewer firms are needed. More surprising is the decrease relative to the rival code case. The reason is that with non-rival, excludable code, the effective supply and value of code is very high. The higher value crowds out capital investment. As can be seen from equation (44), the relatively small number of firms entering is due to a relatively higher effective stock of code and lower stock of capital.

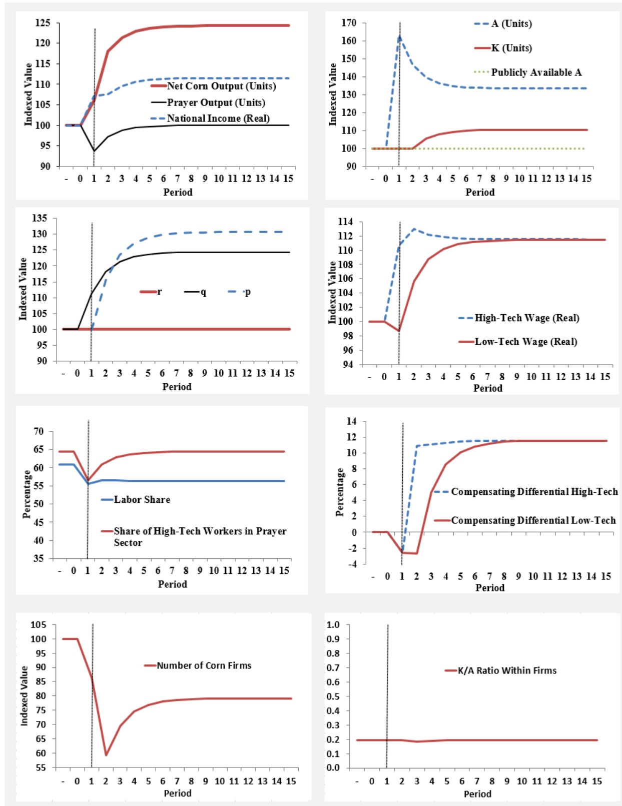


Figure 7: Transition paths based on table 2's parameters and non-rival, excludable code.

6 Empirical Evidence

Relating a two-period deterministic model's predictions to data from a world buffeted by shocks may seem heroic. But, as illustrated in (Hasanhodzic and Kotlikoff 2022), idiosyncratic macro shocks may make little difference to trends arising from deterministic forces such as sustained changes in fiscal policy. And deterministic trends should be evident despite substantial time aggregation. We ask, then, if our model's predictions appear visible, however faintly, in the real world.

Our simulations all feature a temporary rise followed by a decline in labor's share of national income as well as a rise in code as a share of total assets. U.S. labor-share data going back four decades support these trends. Karabarbounis and Neiman (2014) and Brigdman (2014), building on the techniques and findings of Gallin (2002) and Sturgill (2012), are prominent studies making this case. The consensus view is that labor's share has decreased significantly since peaking in the mid 1970's. Armenter (2015) considers the possibility that the decrease in the BLS's measure is driven by the assumption that the proprietors pay themselves the average wage in their industry. When he instead fixes labor's share of proprietor's income at 85 percent, labor' share since 1975 still falls, but by less. There is also recent evidence of a decline in capital per worker, consistent with our model's immiseration scenarios.[21]

Figure 8 displays three measures of labor's share of U.S. income based on three approaches to handing labor's unknown share of proprietorship and partnership income. The orange and gray curves use Bureau of Economic Analysis (BEA) data. The orange curve charts labor's share of total non-proprietorship national income assuming that labor's share of proprietorship income is the same as that of national income.[22] The blue curve displays labor's share of corporate income, i.e., it simply ignores the non-corporate sector.

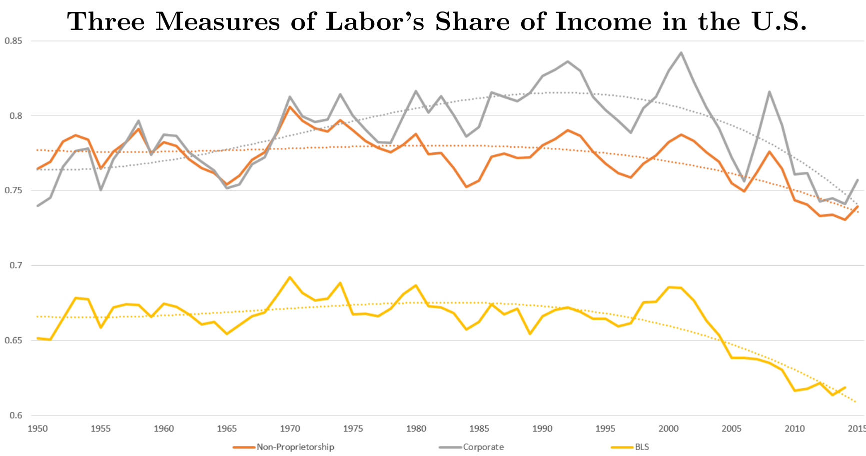


Figure 8: Three measures of the U.S. labor share. The orange curve, labor's share of non-proprietorship income, is calculated as employee compensation divided by national income at producer prices less proprietorship income (NIPA table 1.12, lines 2/(1-25+26-18). The gray curve, labor's share of income in the corporate sector, is calculated as corporate employee compensation divided by corporate business income less corporate taxes net of subsidies (NIPA table 1.13 lines 4/(3-9)). The yellow curve is the BLS's measure of labor share in the private business sector (from the BLS multi-factor productivity series). Dashed lines are fitted third-degree polynomials.

The yellow curve displays labor's share of private businesses including proprietorships as calculated by the Bureau of Labor Statistics (BLS). The BLS imputes labor's share in proprietorship income by assuming proprietors (and partners) earn the annual average wage in their industry. Proprietor income above this amount is considered capital income. This measure is smaller than the others because the BLS's income measure is not net of depreciation.

By all three measures, labor's share of income is lower in 2015 than in the mid 1970's. In the yellow curve, labor's share peaks in the mid-1970s with the two lowest shares recorded in 2014 and 2015. The precise

[21]Capital-hours ratio; BLS multifactor productivity series, Table PG-2-3. Records date back to 1949. Other models, without smart machines, deliver this conclusion. Karabarbounis and Neiman (2014) attribute the decline to capital accumulation and their finding of gross substitutability between capital and labor. Rather than capital abundance, Rognlie (2015) argues that the decrease in the labor share is due to the scarcity of land. He attributes the decline in labors' share to an increase in property values and imputed rents.

[22]National income is measured at producer prices.

percentage-point decline in labor's share between 1975 and 2014 are 5.96 percentage points, 5.88 percentage points, or 4.88 percentage points according to the orange, gray, and yellow curves, respectively.

Our model predicts both a rise in code relative to other economic inputs and an increase in the share of output attributable to intangibles, i.e., inputs that are neither physical capital or labor. Code stocks have certainly increased since the invention of the digital computer and the silicon chip. Figure 9 reports stocks of R&D and software as a share of total U.S. fixed assets. According to the BEA, software grew from essentially zero percent of capital in 1960 to over 1.5 percent today. Combined software and R&D stocks have grown as a share of capital by about 3.5 percentage points over the same period.[23]

The Stock of Software and Software and R&D as a Share of U.S. Fixed Assets

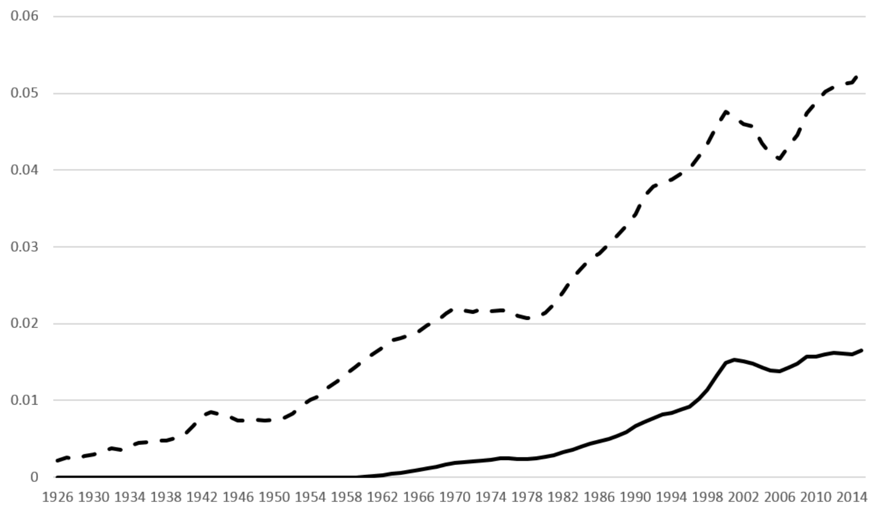


Figure 9: The stock of software (solid line) and software plus R&D assets (dashed line) as a share of total fixed assets (authors' calculation based on NIPA table 2.1).

Many papers suggest that the BEA underestimates the stock of organizational capital and code complementary to computers. Brynjolfson, Hitt, and Yang (2002) find that firms with large investments in computer capital have much higher valuations, that computer-capital investments lead to disproportionately large increases in firm valuations, and that firms that make such investments tend to be more productive in future years. Similarly, Hulten and Hao (2008) find that the book value of R&D-intensive firms in 2006 explains only 31 percent of their valuation. Both these papers argue that only firms who have made large investments in organizational and technological capital are able to implement innovative technologies.

[23] These numbers are likely underestimates of the increasing importance of programmers, scientists and engineers in the economy. Software is decomposed in NIPA table 2.1 into own account, prepackaged and custom software. The true value of prepackaged software in the economy is likely undercounted due to pirating. It is also often free or sold at a discount in order to cross subsidize some other product or subscription (see Parker and Van Alstyne, 2005). BEA estimates of firms' internal creation of their own software are based on very conservative assumptions about the share of programmers who are developing new code, rather than maintaining old code, and the rate at which the software stock decays.

Figure 10 shows the value of the U.S. corporate sector less the replacement cost of its physical and financial assets.[24] This measure of the stock of intangible assets is highly cyclical due to the volatility of the stock market. Despite this, it shows a dramatic secular increase starting in the mid 1970s. For firms in the S&P 500, intangible assets increased from 17 percent of market value in 1975 to 84 percent in 2015 (Ocean Tomo, 2015).

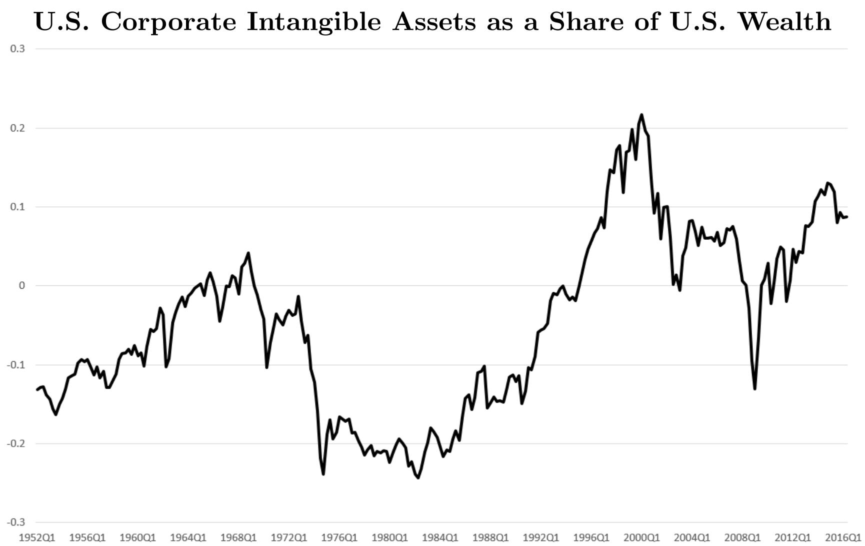


Figure 10: U.S. corporate intangible assets as a share of U.S. wealth is calculated by subtracting the net worth of U.S. corporations from their equity value. Net worth is the replacement cost of fixed assets plus the market value of other assets less liabilities apart from owners' equity. This imputed value of intangible corporate assets (goodwill) is divided by total U.S. wealth (authors' calculation based on Federal Reserve financial accounts series Z.1).

Hall (2001) argues that the increase in the value of economy-wide intangible assets, and therefore Tobin's (average) q, is due to the creation of code and organizational capital within firms, which he calls 'e-capital.' Barkai (2016) and Barkai and Benzell (2018) also note that US firms' output per unit of observed capital has increased even as the marginal cost of capital (as measured by the real interest rate) has decreased since the mid-1980s. Simultaneous decreases in both capital and labor's share of income are consistent with an increasing share for intangibles like code. Barkai (2016) argues that the stock of intangible assets needed to explain the wedge between the observed average product of capital and its marginal cost is implausibly large. The level of intangible assets in 2014 would need to be 42 Trillion (or 54% of U.S. wealth) in order to explain the discrepancy. However, an extremely rapid increase in the share of intangibles in total assets is a phenomena implied by our model.

Long-run immiseration in our model hinges on a long-run decline in capital per worker and a corresponding

[24] U.S. Corporate intangible assets are calculated as U.S. corporate equity less corporate net worth from Federal Reserve series Z.1.

increase in the interest rate. While capital per worker increased at an average rate of 2.5 percent from 1985 to the present, it has been decreasing since 2011 at .5 percent per year on average. This is driven, in part, by a decreasing national net-saving rate in the developed world (Dobrescu et al 2012), also consistent with immiseration in our model. Whether these trends in reduced capital investment continue remains to be seen. But this capital measure significantly underestimates the extent to which physical capital per person has decreased. Capital services as measured by the BLS include accumulation of intellectual property and capital quality increases (through the deflator) that are attributable in our model not to physical capital per worker but to larger stocks of code.

On the other hand, despite low saving and capital investment, and contrary to the prediction of most models of automation, it is the case that real safe interest rates have decreased steadily and dramatically since the mid-1980. However, this does not necessarily indicate that capital itself is over-abundant and its marginal product is low. From 2010 to 2019 the average return to US wealth was 9.5% (Brumm et al 2023).[25]

7 Conclusion

Will smart machines, which are rapidly replacing workers in a wide range of jobs, produce economic misery or prosperity? Our two-period, OLG model admits both outcomes. But it does firmly predict three things - a long-run decline in labor's share of income (apparently underway in OECD countries), tech-booms followed by tech-busts, and a growing dependency of current output on past software investment.

In our simple model, long-run immiseration is caused by a reduction in labor income and, thereby, saving and capital formation in a negative and self-reinforcing cycle. Yes, the economy has better technology. But it has less capital. With the right parameters, the latter factor can outweigh the former. Immiseration is more likely the smaller the propensity to save and the smaller the elasticity of substitution of capital for code.[26] As the appendix shows, our results can be generalized to consider directed technological change in which AI can replace labor or capital. If AI is better at replacing labor, immiseration can readily follow.

[25] Benzell and Brynjolfsson (2019) discuss this riddle. One possible explanation is that the world faces capital abundance, and that capital and labor are gross substitutes. This saving glut hypothesis would be consistent with low interest rates and labor shares, but would not be compatible with a high average return on capital. Other possibilities are that increasing managerial risk aversion (Gormsen and Huber, 2023) or financial frictions have increased the wedge between the safe and risky rate of return on capital. Another possibility is that industrial concentration has increased profits and average returns on capital, while keeping the marginal revenue product of capital investment low (Basu 2019). The explanation favored by Benzell and Brynjolfsson (2019) is that digital abundance has increased scarcity of an inelastically supplied complement to capital, code and ordinary labor.

[26] Alternative intertemporal preferences that make saving of the young depend on the interest rate could also influence the saving rate, making immiseration more (less) likely when saving is a decreasing (increasing) function of the interest rate.

Making higher code retention a win win for all current and future workers as well as initial elderly requires taxing high-skilled workers who benefit from the models technological breakthrough and investing the tax proceeds. This keeps the capital stock and wages from falling. Other policies for managing the rise of smart machines may backfire. For example, policies restricting labor supply will reduce total labor income. While this may temporarily raise wages, it will reduce saving, investment and the capital formation on which wages depend.

To the extent that AI is non rival, countries must weigh several factors in determining whether to grant AI developers property rights to their creations. Requiring new code to be open source, i.e., non excludable, produces a major windfall to initial older generations, a major increase in their consumption, and a major decline in national saving. Ironically, this initiates a transition of capital decline that may eventuate in long-term immiseration. This said, open-source policy increases short-term welfare as well as firm entry. This last implication is an increasingly important desiradatum as countries become concerned about the concentration of power in the hands of a small number of superstar technology companies.

Our simple model illustrates the range of things that smart machines can do for us as well as do to us. Its central message is disturbing. Absent appropriate fiscal policy, which redistributes from winners to losers, smart machines can mean collective long-term misery. Ironically, the same AI that helps us produce more in the present can limit our production in the future.

Appendix

A Directed Technological Change

Recent models of technological change, e.g., Acemoglu (2002) and Acemoglu and Restrepo (2018), emphasize the directedness of technological change. Directedness references the ability to produce technologies that complement an array of productive inputs. Directedness can readily be included in our model. To illustrate, consider a one-sector version of our model in which high-tech workers can produce labor-substituting code, capital-substituting code, or both.

Production of corn satisfies

$$Y_t = D((1-\gamma)(W_t + A_t^L)^\alpha + \gamma(K_t + A_t^K)^\alpha)^{\frac{1}{\alpha}}. \tag{52}$$

This production function deviates from the baseline model in two ways. First, we introduce a type of labor, W, that is a perfect substitute for software but that doesn't create software as a byproduct. Second, we add a type of software A^K which is a substitute for capital.

Producers face the following profit maximization problem.

$$\pi_t = Y_t(W_t + A_t^L, K_t + A_t^K) - w_t W_t - r_t K_t - A_t^L m_t^L - A_t^K m_t^K, \tag{53}$$

where w_t is the wage of non-coders and m_t^L and m_t^K are the rental prices of labor substituting and capital substituting software, respectively.

Both types of code are written by software companies with fixed costs of entry and decreasing returns to scale. Production of code satisfies

$$A_{t,i}^L = z_L(L_{t,i}^L)^{\beta_L} \tag{54}$$

$$A_{t,j}^K = z_K(L_{t,j}^K)^{\beta_K}, \tag{55}$$

where $L_{t,i}^L$ and $L_{t,j}^K$ are demands for coders by labor substituting and capital substituting software companies, respectively and z_K, z_L, β_L and β_K are parameters.

Software firms maximize profits – revenues from renting code net of paying coders and covering each period's fixed cost of operation, F^L or F^K. Firms rent out their software at its marginal product and then sell their software after depreciation at prices p_t^L and p_t^K.

$$\pi_{t,i}^L = A_{t,i}^L(m_t^L + \delta^L p_t^L) - w_t L_{t,i}^L - F_t^L \tag{56}$$

$$\pi_{t,j}^K = A_{t,j}^K(m_t^K + \delta^K p_t^K) - w_t L_{t,j}^K - F_t^K \tag{57}$$

Total software of each type accumulates as in the baseline model.

$$A_t^L = \sum A_{t,i}^L + \delta_L A_{t-1}^L \tag{58}$$

$$A_t^K = \sum A_{t,i}^K + \delta_K A_{t-1}^K. \tag{59}$$

Firms enter until profits are zero. This implies

$$\pi_{t,i}^L = 0 \tag{60}$$

$$\pi_{t,i}^K = 0 \tag{61}$$

Combining equations gives

$$0 = z_L(L_t^L/N_t^L)^{\alpha_L}(m_t^L + \delta^L p_t^L) - w_t L_t^1/N_t^L - F_t^L \tag{62}$$

$$0 = z_K(L_t^K/N_t^K)^{\alpha_K}(m_t^K + \delta^K p_t^K) - w_t K_t^1/N_t^K - F_t^K, \tag{63}$$

where N_t^K and N_t^L are the number of K and L software companies, respectively.

Both types of software are priced as the present discounted value of their marginal product. The capital stock equals saving of the young net of their purchase of ownership rights to software.

$$K_{t+1} = \phi I_t - p_t^L \delta A_t^L - p_t^K \delta A_t^K \tag{64}$$

Labor market equilibrium requires

$$\overline{L} = L_t^L + L_t^K + W_t. \tag{65}$$

Since workers can move freely between the tasks of ordinary worker, capital-substituting coder, or labor substituting coder, the wage, w_t, is the same across all tasks. Hence, the total income of the young is

$$I_t = w_t \overline{L}. \tag{66}$$

There is no depreciation of capital. Output is either consumed, invested or used to cover fixed costs.

$$Y_t = C_{y,t} + C_{o,t} + K_{t+1} - K_t - F_t^K N_t^K - F_t^L N_t^L, \tag{67}$$

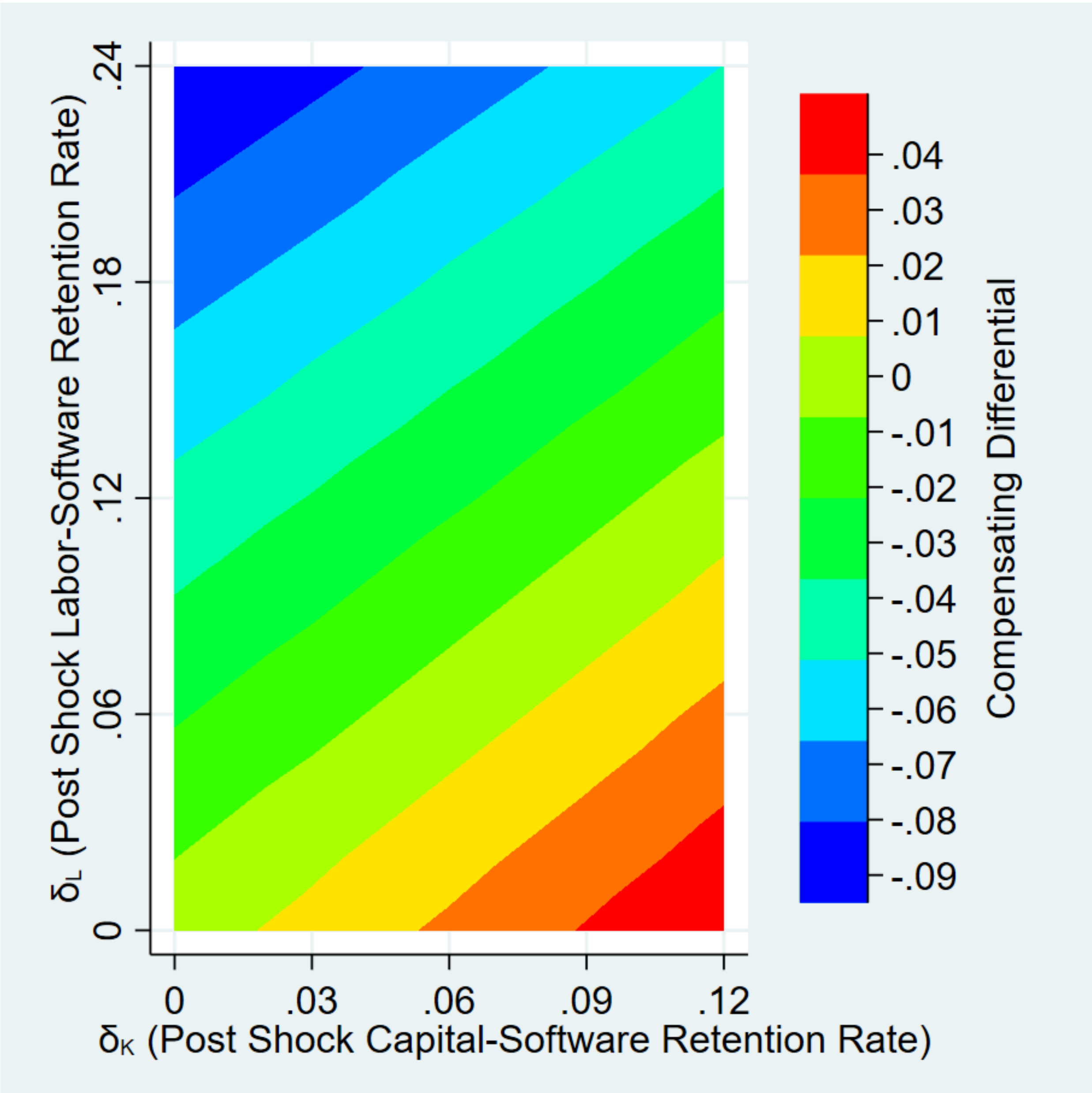
Appendix figure 1 reports steady-state results based on the parameters in appendix table 1. As can be seen, steady-state outcomes for an economy with a higher δ^L are worse, while those with relatively higher δ^K are better. The mechanisms for immiseration are similar to our baseline model. The increase in A^L stocks decreases wages and increases interest rates. The decrease in wages decreases capital, which decreases wages yet further. Immiseration can arise if the exogenous shock makes it disproportionately easier to accumulate labor- rather than capital-substituting code.

Appendix Table 1
Parameters for Directed Technical Change Simulation

Model Parameter	Role	Value
α	Elasticity Param. in Final Good Prod.	.952
γ	Capital Input Share Param.	.7
$\overline{L}$	Total Labor Supply	25
δ^L	Labor Subs. Code Retention Rate	0 initially
δ^K	Capital Subs. Code Retention Rate	0 initially
ϕ	Saving Rate	.8
F^L	L Code Firm Fixed Cost	$5 * 10^{-13}$
F^K	K Code Firm Fixed Cost	$5 * 10^{-13}$
Z_L	L Code Productivity Param.	.00013
Z_K	K Code Productivity Param.	.00013
β_L	L Decreasing Returns to L-Software Coding Param.	.6
β_K	K Decreasing Returns to K-Software Coding Param.	.6
D	TFP in Final Production	1

Appendix Table 1: This table presents parameter values for illustrations of the effects of a one-time, permanent increase in the depreciation rate of labor substituting and capital substituting software as displayed in appendix figure 1.

Appendix Figure 1
Long-Run Compensating Differential for Alternative δ Shocks



Compensating Differential references the ratio of consumption needed to achieve lifetime utility of households in the long run to initial steady-state consumption less 1. Parameters not on axes are given in appendix table 1.

B Derivation Details for Cobb Douglas Steady State

In this section, we present the analytical derivations for the case when the production functions follow the Cobb-Douglas form. We include step by step all the expressions that lead to the steady state. Then, we show how to obtain $\frac{dk}{d\delta}$ from the steady state equation.

B.1 The Set of Equations Solving the Model

Two conditions come directly from the optimization problem of households described in Chapter 3:

- **Savings**

$$K_{t+1} + p_t\delta A_t = \phi[z(m_t + p_t\delta)h_t + w_t^H H_t + w_t^L L_t] \tag{68}$$

- **No arbitrage on returns on their assets**

$$1 + r_{t+1} = \frac{p_{t+1}\delta^2 A_t + m_{t+1}\delta A_t}{p_t\delta A_t}. \tag{69}$$

Firms maximize profits using Cobb-Douglas production functions:

$$S_t = D_s H_t^{\gamma} L_t^{1-\gamma}, \tag{70}$$

and

$$Y_t = D_y K_t^{\alpha} A_t^{1-\alpha}. \tag{71}$$

The marginal products resulting, respectively, are

$$w_t^L = q_t(1-\gamma)D_s H_t^{\gamma} L_t^{-\gamma}, \tag{72}$$

$$w_t^H = q_t\gamma D_s H_t^{\gamma-1} L_t^{1-\gamma}, \tag{73}$$

$$m_t = (1-\alpha)D_y K_t^{\alpha} A_t^{-\alpha}, \tag{74}$$

and

$$r_t = \alpha D_y K_t^{\alpha-1} A_t^{1-\alpha}. \tag{75}$$

Additionally, high-tech workers should not encounter any arbitrage opportunities that would entice them to switch sectors:

- **No arbitrage on earnings**

$$w_t^H = z(m_t + p_t\delta). \tag{76}$$

Finally, the equilibrium should take into account how code accumulates overtime and ensure all markets clear:

- **Code accumulation rule**

$$A_t = A_{t-1}\delta + zh_t. \tag{77}$$

- **Supply-Demand Identities**

$$D_y K_t^{\alpha} A_t^{1-\alpha} = C_{y,t}^H + C_{o,t}^H + C_{y,t}^L + C_{o,t}^L + K_{t+1} - K_t, \tag{78}$$

$$H_t = H_{A,t} + H_{S,t}, \tag{79}$$

and

$$D_s H_t^{\gamma} L_t^{1-\gamma} = S_{y,t}^H + S_{o,t}^H + S_{y,t}^L + S_{o,t}^L. \tag{80}$$

B.2 Solving for the Steady State

We first establish the set of equations that will help us obtain the steady state of $k = \frac{K}{A}$:

- **Savings** from (68)

$$K + p\delta A = \phi[z(m + p\delta)h + w^H H + w^L L]. \tag{81}$$

- **No arbitrage on capital returns** from (69)

$$1 + r = \frac{p\delta^2 A + m\delta A}{p\delta A}. \tag{82}$$

- **No arbitrage on earnings** from (76)

$$w^H = z(m + p\delta). \tag{83}$$

- **Code accumulation rule** from (77)

$$A = A\delta + zh. \tag{84}$$

- **Prices** from (72)-(75)

$$w^L = q(1-\gamma)D_s H^\gamma L^{-\gamma}, \tag{85}$$

$$w^H = q\gamma D_s H^{\gamma-1} L^{1-\gamma}, \tag{86}$$

$$m = (1-\alpha)D_y K^\alpha A^{-\alpha}, \tag{87}$$

  and

$$r = \alpha D_y K^{\alpha-1} A^{1-\alpha}. \tag{88}$$

- **Supply-Demand Identities** from (78) and (80)

$$D_y K^\alpha A^{1-\alpha} = C_y^H + C_o^H + C_y^L + C_y^L, \tag{89}$$

  and

$$D_s H^\gamma L^{1-\gamma} = S_y^H + S_o^H + S_y^L + S_o^L. \tag{90}$$

Now consider the following steps:

1. Step 1. Use the no arbitrage condition (82) and solve for p :

$$p = \frac{m}{1+r-\delta}. \tag{91}$$

2. Step 2. Now replacing m and r with (87) and (88), respectively, we get

$$p = \frac{(1-\alpha)D_y K^\alpha A^{-\alpha}}{1+\alpha D_y K^{\alpha-1} A^{1-\alpha} - \delta}. \tag{92}$$

3. Step 3. Now consider equation (81). Use the no arbitrage on earnings condition (83) to obtain

$$K + p\delta A = \phi[w^H(h+H) + w^L L]. \tag{93}$$

   In equilibrium, overall labor supply of low-tech workers is $\overline{L}=L$ and high-tech labor supply is $h+H=\overline{H}$, both constant. So we can re-express (93) as

$$K + p\delta A = \phi[w^H \overline{H} + w^L \overline{L}]. \tag{94}$$

4. Step 4. Now consider equation (89). Once we substitute each type of consumption with their corresponding demand functions, we obtain

$$\begin{aligned} D_y K^\alpha A^{1-\alpha} &= (1-\phi)(1-\kappa)[z(m+p\delta)h + w^H H + w^L L] \\ &+ \phi(1-\kappa)[z(m+p\delta)h + w^H H + w^L L][1+r]. \end{aligned} \tag{95}$$

5. Step 5. Furthermore, we can use (94) to simplify:

$$D_y K^\alpha A^{1-\alpha} = (1-\phi)(1-\kappa)[\frac{K+p\delta A}{\phi}] + \phi(1-\kappa)[\frac{K+p\delta A}{\phi}][1+r]. \tag{96}$$

6. Step 6. We now rearrange parameters and re-write the above equation in terms of $k = \frac{K}{A}$. Thus, we have the following expression:

$$D_y k^\alpha = \frac{(1-\phi)(1-\kappa)}{\phi}[k+p\delta] + (1-\kappa)[k+p\delta][1+r]. \tag{97}$$

7. Step 7. Finally, we can substitute p and r to get an expression with only one variables, k, and several constant parameters, together defining an steady state:

$$\begin{aligned} D_y k^\alpha &= \frac{(1-\phi)(1-\kappa)}{\phi}[k + \frac{(1-\alpha)D_y k^\alpha \delta}{1+\alpha D_y k^{\alpha-1} - \delta}] \\ &+ (1-\kappa)[k + \frac{(1-\alpha)D_y k^\alpha \delta}{1+\alpha D_y k^{\alpha-1} - \delta}][1+\alpha D_y k^{\alpha-1}]. \end{aligned} \tag{98}$$

B.3 Obtaining the Derivative $\frac{dk}{d\delta}$

As you can observe from the previous equation, it is impossible to find an explicit expression that directly relates k to the model parameters. Specifically, if we want to establish the relationship between k and delta, $\frac{dk}{d\delta}$, it is necessary to employ the implicit function. theorem The following steps outline the process leading to such an expression.

1. Step 1: We first group all terms similar to Dyk^{α}. To do so we expand the right hand side (RHS) of (98):

$$D_y k^{\alpha} = \left[\frac{(1-\phi)(1-\kappa)}{\phi} + (1-\kappa)\right]k + \left[\frac{(1-\phi)(1-\kappa)}{\phi}\right]\frac{(1-\alpha)D_y k^{\alpha}\delta}{1+\alpha D_y k^{\alpha-1}-\delta}$$

$$+(1-\kappa)k\alpha Dyk^{\alpha-1} + \frac{(1-\kappa)(1-\alpha)D_y k^{\alpha}\delta + (1-\kappa)(1-\alpha)\delta Dyk^{\alpha}\alpha Dyk^{\alpha-1}}{1+\alpha Dyk^{\alpha-1}-\delta}. \quad (99)$$

2. Step 2: We define $A = \frac{(1-\phi)(1-\kappa)}{\phi}$, and with further simplification we obtain:

$$D_y k^{\alpha} = (A + (1-\kappa))k + \frac{A(1-\alpha)D_y k^{\alpha}\delta}{1+\alpha D_y k^{\alpha-1}-\delta}$$

$$+(1-\kappa)k\alpha Dyk^{\alpha} + \frac{(1-\kappa)(1-\alpha)D_y k^{\alpha}\delta + (1-\kappa)(1-\alpha)\delta Dyk^{\alpha}\alpha Dyk^{\alpha-1}}{1+\alpha Dyk^{\alpha-1}-\delta}. \quad (100)$$

3. Step 3: Now send to the LHS of all terms similar to Dyk^{α} and by grouping terms we get a shorter expression:

$$\left[1-(1-\kappa)\alpha - \frac{B}{1+\alpha Dyk^{\alpha-1}-\delta}\right]D_y k^{\alpha}$$

$$= \frac{(1-\kappa)}{\phi}k + \frac{(1-\kappa)(1-\alpha)\delta\alpha Dy^2 k^{2\alpha-1}}{1+\alpha Dyk^{\alpha-1}-\delta}, \quad (101)$$

where $B = \frac{(1-\alpha)(1-\kappa)\delta}{\phi}$.

4. Step 4: Now we multiply both sides of the equation times $(1+\alpha Dyk^{\alpha-1}-\delta)$, to obtain

$$[(1-(1-\kappa)\alpha)(1-\delta) - B]D_y k^{\alpha} + (\alpha Dy^2 k^{2\alpha-1})(1-(1-\kappa)\alpha) =$$

$$= \frac{(1-\kappa)(1-\delta)}{\phi}k + \frac{(1-\kappa)}{\phi}\alpha Dyk^{\alpha} + (1-\kappa)(1-\alpha)\delta\alpha Dy^2 k^{2\alpha-1}. \quad (102)$$

5. Step 8: Placing similar terms together in both sides of the equation:

$$[(1-(1-\kappa)\alpha)(1-\delta) - B - \frac{(1-\kappa)}{\phi}\alpha]D_y k^{\alpha} =$$

$$= \frac{(1-\kappa)(1-\delta)}{\phi}k + (1-\kappa)(1-\alpha)\delta\alpha Dy^2 k^{2\alpha-1} - (\alpha Dy^2 k^{2\alpha-1})(1-(1-\kappa)\alpha). \tag{103}$$

6. Step 9: Dividing both sides of the equation by k^α and grouping:

$$[(1-(1-\kappa)\alpha)(1-\delta) - B - \frac{(1-\kappa)}{\phi}\alpha]D_y =$$

$$= \frac{(1-\kappa)(1-\delta)}{\phi}k^{1-\alpha} + [(1-\kappa)(1-\alpha)\delta\alpha - \alpha(1-(1-\kappa)\alpha)]Dy^2 k^{\alpha-1}. \tag{104}$$

7. Step 10: We use the Implicit Function Theorem to get $\frac{dk}{d\delta}$=-$\frac{F_\delta}{F_k}$. The following are the resulting partial derivatives:

$$F_\delta =$$

$$-\left[-(1-(1-\kappa)\alpha) - \frac{(1-\kappa)(1-\alpha)}{\phi}\right]D_y - \frac{(1-\kappa)}{\phi}k^{1-\alpha} + (1-\kappa)(1-\alpha)\alpha D_y^2 k^{\alpha-1} \tag{105}$$

$$F_k =$$

$$\frac{(1-\alpha)(1-\kappa)(1-\delta)}{\phi}k^{-\alpha} - (1-\alpha)[(1-\kappa)(1-\alpha)\delta\alpha - \alpha(1-(1-\kappa)\alpha)]D_y^2 k^{\alpha-2} \tag{106}$$

8. Step 11: Finally, the derivative would take the form:

$$\frac{dk}{d\delta} = -\frac{F_\delta}{F_k} = -$$

$$\frac{-\left[-(1-(1-\kappa)\alpha) - \frac{(1-\kappa)(1-\alpha)}{\phi}\right]D_y - \frac{(1-\kappa)}{\phi}k^{1-\alpha} + (1-\kappa)(1-\alpha)\alpha D_y^2 k^{\alpha-1}}{\frac{(1-\alpha)(1-\kappa)(1-\delta)}{\phi}k^{-\alpha} - (1-\alpha)[(1-\kappa)(1-\alpha)\delta\alpha - \alpha(1-(1-\kappa)\alpha)]D_y^2 k^{\alpha-2}} \tag{107}$$

C Sensitivity Analysis Table

Appendix Table 2 Sensitivity Analysis

Baseline

					Labor					National	Output							Wage			High-Tech	Compensating Differentials	
Period	ϵ_s	ϵ_y	δ	ϕ	High-Tech	Low-Tech	κ	γ	z	Income	Corn	Prayers	K	A	p	q	r	Low-Tech	High-Tech	Labor_Share	Coding	High-Tech	Low-Tech
Initial Steady State	1.0	1.0	**0.0**	0.5	1.0	1.0	0.5	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	50.0	100.0	75.0	50.0	0.0	0.0
1	1.0	1.0	**0.5**	0.5	1.0	1.0	0.5	0.5	1.0	107	106.1	93.5	100.0	112.6	100.0	116.3	106.1	43.7	124.4	75.3	56.3	-2.5	-2.5
2	1.0	1.0	**0.5**	0.5	1.0	1.0	0.5	0.5	1.0	112	117.5	106.4	96.5	143.0	83.9	116.2	121.7	56.7	109.2	66.1	43.3	20.3	5.2
3	1.0	1.0	**0.5**	0.5	1.0	1.0	0.5	0.5	1.0	112	116.0	109.1	88.3	152.4	80.3	110.5	131.4	59.5	101.3	64.0	40.5	8.8	23.3
4	1.0	1.0	**0.5**	0.5	1.0	1.0	0.5	0.5	1.0	111	113.2	109.6	82.1	156.0	78.0	106.0	137.8	60.1	96.6	63.3	39.9	4.5	24.4
10	1.0	1.0	**0.5**	0.5	1.0	1.0	0.5	0.5	1.0	110	111.0	109.7	78.1	157.6	74.3	102.9	142.1	60.2	93.8	63.0	39.8	2.3	23.0
Steady State	1.0	1.0	**0.5**	0.5	1.0	1.0	0.5	0.5	1.0	108	107.0	109.6	71.6	159.8	74.1	97.8	149.4	60.0	89.2	62.5	40.0	-1.0	18.8

Prayer Taste Shock κ from 0.5 to 0.25

					Labor					National	Output							Wage			High-Tech	Compensating Differentials	
Period	ϵ_s	ϵ_y	δ	ϕ	High-Tech	Low-Tech	κ	γ	z	Income	Corn	Prayers	K	A	p	q	r	Low-Tech	High-Tech	Labor_Share	Coding	High-Tech	Low-Tech
Initial Steady State	1.0	1.0	0.0	0.5	1.0	1.0	**0.25**	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	25.0	100.0	62.5	75.0	0.0	0.0
1	1.0	1.0	0.5	0.5	1.0	1.0	**0.25**	0.5	1.0	107	101.7	94.7	100.0	103.4	100.0	111.1	101.7	22.4	117.3	62.8	77.6	-0.8	-0.8
2	1.0	1.0	0.5	0.5	1.0	1.0	**0.25**	0.5	1.0	112	114.1	112.9	91.4	142.5	55.3	107.1	124.9	31.9	94.9	49.7	68.1	22.4	9.8
3	1.0	1.0	0.5	0.5	1.0	1.0	**0.25**	0.5	1.0	108	108.7	118.0	74.8	158.2	49.9	96.0	145.4	34.8	81.3	46.2	65.2	5.7	34.8
4	1.0	1.0	0.5	0.5	1.0	1.0	**0.25**	0.5	1.0	105	102.6	119.6	63.9	164.8	46.9	88.0	160.6	35.7	73.6	44.9	64.3	-4.1	33.5
10	1.0	1.0	0.5	0.5	1.0	1.0	**0.25**	0.5	1.0	102	98.2	120.0	57.5	167.7	43.2	83.1	170.8	36.0	69.2	44.3	64.0	-9.8	28.9
Steady State	1.0	1.0	0.5	0.5	1.0	1.0	**0.25**	0.5	1.0	98	91.6	120.1	49.3	170.5	43.0	76.3	186.0	36.1	63.6	43.8	63.9	-17.3	19.2

Prayer Taste Shock κ from 0.5 to 0.75

					Labor					National	Output							Wage			High-Tech	Compensating Differentials	
Period	ϵ_s	ϵ_y	δ	ϕ	High-Tech	Low-Tech	κ	γ	z	Income	Corn	Prayers	K	A	p	q	r	Low-Tech	High-Tech	Labor_Share	Coding	High-Tech	Low-Tech
Initial Steady State	1.0	1.0	0.0	0.5	1.0	1.0	**0.75**	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	75.0	100.0	87.5	25.0	0.0	0.0
1	1.0	1.0	0.5	0.5	1.0	1.0	**0.75**	0.5	1.0	106	119.0	92.8	100.0	141.5	100.0	120.9	119.0	64.6	130.3	86.9	35.4	-4.9	-4.9
2	1.0	1.0	0.5	0.5	1.0	1.0	**0.75**	0.5	1.0	109	130.6	101.1	103.8	164.3	119.0	127.0	125.8	76.6	125.6	81.9	23.4	14.1	-1.7
3	1.0	1.0	0.5	0.5	1.0	1.0	**0.75**	0.5	1.0	109	132.8	102.4	105.1	167.7	119.4	128.4	126.3	78.6	125.5	81.2	21.4	7.6	10.0
4	1.0	1.0	0.5	0.5	1.0	1.0	**0.75**	0.5	1.0	110	133.4	102.6	105.9	168.0	119.7	129.1	126.0	79.0	125.8	81.2	21.0	6.8	11.9
10	1.0	1.0	0.5	0.5	1.0	1.0	**0.75**	0.5	1.0	110	133.6	102.7	106.4	167.8	120.5	129.5	125.6	79.0	126.2	81.2	21.0	6.7	12.4
Steady State	1.0	1.0	0.5	0.5	1.0	1.0	**0.75**	0.5	1.0	110	134.1	102.7	107.6	167.2	120.7	130.5	124.7	79.1	127.1	81.2	20.9	6.9	12.7

Low Saving Rate f = 0.1

					Labor					National	Output							Wage			High-Tech	Compensating Differentials	
Period	ϵ_s	ϵ_y	δ	f	High-Tech	Low-Tech	κ	γ	z	Income	Corn	Prayers	K	A	p	q	r	Low-Tech	High-Tech	Labor_Share	Coding	High-Tech	Low-Tech
Initial Steady State	1.0	1.0	0.0	**0.1**	1.0	1.0	0.5	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	50.0	100.0	75.0	50.0	0.0	0.0
1	1.0	1.0	0.5	**0.1**	1.0	1.0	0.5	0.5	1.0	102	101.2	98.8	100.0	102.3	100.0	105.6	101.2	48.8	106.9	75.4	51.2	-0.1	-0.1
2	1.0	1.0	0.5	**0.1**	1.0	1.0	0.5	0.5	1.0	104	100.8	110.5	78.7	129.0	47.4	93.2	128.0	61.1	84.4	65.5	38.9	6.7	4.3
3	1.0	1.0	0.5	**0.1**	1.0	1.0	0.5	0.5	1.0	101	93.2	113.1	63.6	136.5	43.9	83.4	146.5	64.0	73.7	63.4	36.0	-9.4	10.7
4	1.0	1.0	0.5	**0.1**	1.0	1.0	0.5	0.5	1.0	99	88.0	113.8	55.7	138.8	42.2	77.9	157.8	64.7	68.4	62.8	35.3	-15.9	7.6
10	1.0	1.0	0.5	**0.1**	1.0	1.0	0.5	0.5	1.0	98	84.9	113.9	51.7	139.6	40.3	74.8	164.4	64.9	65.7	62.6	35.1	-19.0	4.8
Steady State	1.0	1.0	0.5	**0.1**	1.0	1.0	0.5	0.5	1.0	96	81.4	114.0	47.3	140.2	40.2	71.5	172.2	64.9	62.7	62.5	35.1	-22.4	0.8

High Saving Rate f = 0.9

					Labor					National	Output							Wage			High-Tech	Compensating Differentials	
Period	ϵ_s	ϵ_y	δ	f	High-Tech	Low-Tech	κ	γ	z	Income	Corn	Prayers	K	A	p	q	r	Low-Tech	High-Tech	Labor_Share	Coding	High-Tech	Low-Tech
Initial Steady State	1.0	1.0	0.0	**0.9**	1.0	1.0	0.5	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	50.0	100.0	75.0	50.0	0.0	0.0
1	1.0	1.0	0.5	**0.9**	1.0	1.0	0.5	0.5	1.0	111	110.2	88.6	100.0	121.5	100.0	117.7	110.2	39.2	132.9	74.3	60.8	-4.7	-4.7
2	1.0	1.0	0.5	**0.9**	1.0	1.0	0.5	0.5	1.0	117	126.9	103.2	104.4	154.3	106.5	126.1	121.6	53.2	122.3	65.6	46.8	25.0	-1.9
3	1.0	1.0	0.5	**0.9**	1.0	1.0	0.5	0.5	1.0	118	129.4	106.3	102.0	164.1	104.4	125.2	126.8	56.6	117.7	63.8	43.4	16.4	23.9
4	1.0	1.0	0.5	**0.9**	1.0	1.0	0.5	0.5	1.0	117	128.9	107.0	99.2	167.4	102.9	123.2	129.9	57.3	115.1	63.2	42.7	13.7	28.6
10	1.0	1.0	0.5	**0.9**	1.0	1.0	0.5	0.5	1.0	117	128.0	107.2	97.0	168.9	100.1	121.5	131.9	57.4	113.4	62.9	42.6	12.4	28.8
Steady State	1.0	1.0	0.5	**0.9**	1.0	1.0	0.5	0.5	1.0	116	125.7	107.0	92.4	171.0	99.7	117.7	136.0	57.2	110.0	62.5	42.8	10.5	26.5

Sensitivity Analysis. In all examples, the δ is shocked from 0 in the initial steady state to .5. Simulations subsequent to the baseline have one (highlighted) parameter changed. Output of both products are in units, not at market prices. All endogenous variables are indexed.

Appendix Table 2 Sensitivity Analysis (Continued)

Perfect Substitutability in Prayer Sector $\epsilon_s = \infty$

Period	ϵ_s	ϵ_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	National Income	Output Corn	Output Prayers	K	A	p	q	r	Wage Low-Tech	Wage High-Tech	Labor_Share	High-Tech Coding	Compensating Differentials High-Tech	Compensating Differentials Low-Tech
Initial Steady State	∞	1.0	0.0	0.5	1.0	1.0	0.5	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	100.0	100.0	75.0	66.7	0.0	0.0
1	∞	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	108	109.3	90.3	100.0	119.4	100.0	122.8	109.3	100.0	122.8	75.2	79.6	-3.2	-3.2
2	∞	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	113	116.1	111.0	97.9	137.7	85.8	112.2	118.6	100.0	112.2	65.4	52.0	17.6	17.6
3	∞	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	111	111.3	112.9	86.6	143.0	81.4	103.6	128.5	100.0	103.6	63.9	49.4	14.1	14.1
4	∞	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	109	107.3	112.9	79.0	145.6	78.7	98.1	135.8	100.0	98.1	63.3	49.4	10.3	10.3
10	∞	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	108	104.6	112.8	74.4	147.2	74.4	94.7	140.7	100.0	94.7	63.0	49.6	7.8	7.8
Steady State	∞	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	106	100.3	112.5	67.2	149.8	74.1	89.3	149.4	100.0	89.3	62.5	50.0	3.6	3.6

Strong Complementarity in Prayer Sector $\epsilon_s = 1.0$

Period	ϵ_s	ϵ_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	National Income	Output Corn	Output Prayers	K	A	p	q	r	Wage Low-Tech	Wage High-Tech	Labor_Share	High-Tech Coding	Compensating Differentials High-Tech	Compensating Differentials Low-Tech
Initial Steady State	**1.0**	1.0	0.0	0.5	1.0	1.0	0.5	0.5	1.0	100	100.0	100.0	100.0	100.0	-	100.0	100.0	31.5	100.0	75.0	43.8	0.0	0.0
1	**1.0**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	107	106.0	93.6	100.0	112.3	100.0	116.0	106.0	25.8	124.4	75.3	49.2	-2.4	-2.4
2	**1.0**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	112	117.7	106.1	96.5	143.6	83.7	116.6	122.0	38.0	108.8	66.1	38.3	20.4	-1.6
3	**1.0**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	112	116.6	108.6	88.5	153.7	80.1	111.5	131.8	41.1	101.0	64.0	35.9	8.3	30.5
4	**1.0**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	111	114.1	109.2	82.6	157.6	77.9	107.2	138.1	41.8	96.4	63.3	35.4	3.7	35.1
10	**1.0**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	110	111.9	109.2	78.7	159.3	74.3	104.2	142.3	41.9	93.6	62.9	35.3	1.5	34.4
Steady State	**1.0**	1.0	0.5	0.5	1.0	1.0	0.5	0.5	1.0	109	108.0	109.1	72.3	161.4	74.1	99.1	149.4	41.7	89.2	62.5	35.4	-1.7	30.0

Strong Complementarity in Corn Sector $e\epsilon_y = 0.8$

Period	ϵ_s	ϵ_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	National Income	Output Corn	Output Prayers	K	A	p	q	r	Wage Low-Tech	Wage High-Tech	Labor_Share	High-Tech Coding	Compensating Differentials High-Tech	Compensating Differentials Low-Tech
Initial Steady State	1.0	**0.8**	0.0	0.5	1.0	1.0	0.5	0.5	1.0	100.0	100.0	100.0	100.0	100.0	-	100.0	100.0	52.1	100.0	72.9	47.9	0.0	0.0
1	1.0	**0.8**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	106.1	105.0	94.7	100.0	111.3	100.0	113.4	106.3	46.7	119.8	72.9	53.3	-1.8	-1.8
2	1.0	**0.8**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	110.4	113.1	108.2	96.7	137.1	74.6	110.5	121.5	61.0	102.1	64.0	39.0	18.4	6.1
3	1.0	**0.8**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	109.1	109.1	111.1	87.9	143.1	69.8	102.9	131.1	64.3	92.7	62.1	35.7	5.3	23.3
4	1.0	**0.8**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	107.2	104.4	111.7	80.8	144.6	66.6	96.8	137.8	65.1	86.6	61.4	34.9	0.0	23.3
10	1.0	**0.8**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	105.5	100.7	111.9	75.7	144.9	59.6	92.3	142.8	65.2	82.5	61.0	34.8	-3.1	20.9
Steady State	1.0	**0.8**	0.5	0.5	1.0	1.0	0.5	0.5	1.0	101.5	92.3	111.9	65.2	145.0	58.5	82.7	154.3	65.3	73.9	60.3	34.7	-9.3	13.6

Strong Substitutability in Corn Sector $\epsilon_y = 10$

Period	ϵ_s	ϵ_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	z	National Income	Output Corn	Output Prayers	K	A	p	q	r	Wage Low-Tech	Wage High-Tech	Labor_Share	High-Tech Coding	Compensating Differentials High-Tech	Compensating Differentials Low-Tech
Initial Steady State	1.0	**10.0**	0.0	0.5	1.0	1.0	0.5	0.5	1.0	100.0	100.0	100.0	100.0	100.0	-	100.0	100.0	45.8	100.0	79.6	54.2	0.0	0.0
1	1.0	**10.0**	0.7	0.5	1.0	1.0	0.5	0.5	1.0	122.2	114.9	83.7	100.0	125.3	100.0	149.2	101.4	32.0	178.3	82.8	68.0	-9.2	-9.2
2	1.0	**10.0**	0.7	0.5	1.0	1.0	0.5	0.5	1.0	130.9	147.9	98.1	87.7	190.9	160.7	170.7	105.4	44.0	174.1	72.0	56.0	42.5	-0.3
3	1.0	**10.0**	0.7	0.5	1.0	1.0	0.5	0.5	1.0	133.0	160.4	101.4	63.2	231.3	156.9	173.2	109.8	47.0	170.8	68.9	53.0	35.0	29.8
4	1.0	**10.0**	0.7	0.5	1.0	1.0	0.5	0.5	1.0	133.7	166.8	102.5	44.2	257.7	154.0	172.5	114.2	48.0	168.4	67.3	52.0	33.1	36.7
10	1.0	**10.0**	0.7	0.5	1.0	1.0	0.5	0.5	1.0	134.2	170.8	103.1	31.7	275.2	148.0	171.7	118.4	48.6	166.6	66.3	51.4	32.3	38.9
Steady State	1.0	**10.0**	0.7	0.5	1.0	1.0	0.5	0.5	1.0	136.4	178.0	104.5	12.9	304.8	147.6	170.6	130.0	50.0	163.3	64.6	50.0	31.4	43.2

Code Writing Productivity Shock z from 1.0 to 1.5

Period	ϵ_s	ϵ_y	δ	ϕ	Labor High-Tech	Labor Low-Tech	κ	γ	**z**	National Income	Output Corn	Output Prayers	K	A	p	q	r	Wage Low-Tech	Wage High-Tech	Labor_Share	High-Tech Coding	Compensating Differentials High-Tech	Compensating Differentials Low-Tech
Initial Steady State	1.0	1.0	0.0	0.5	1.0	1.0	0.5	0.5	**1.0**	100.0	100.0	100.0	100.0	100.0	-	100.0	100.0	50.0	100.0	75.0	50.0	0.0	0.0
1	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	**1.5**	119.1	132.7	90.9	100.0	176.2	100.0	136.0	132.7	41.3	149.7	74.1	58.7	-1.5	-1.5
2	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	**1.5**	130.5	157.7	105.3	112.3	221.7	76.8	149.6	140.5	55.5	142.1	65.0	44.5	35.1	11.6
3	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	**1.5**	132.8	162.2	108.6	112.4	234.0	75.8	150.4	144.3	59.0	138.5	63.3	41.0	25.9	39.6
4	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	**1.5**	133.1	162.3	109.4	110.9	237.6	75.2	149.4	146.4	59.8	136.6	62.8	40.2	23.3	45.3
10	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	**1.5**	133.0	161.7	109.5	109.5	238.8	74.1	148.3	147.7	60.0	135.4	62.7	40.0	22.2	46.2
Steady State	1.0	1.0	0.5	0.5	1.0	1.0	0.5	0.5	**1.5**	132.4	160.1	109.5	106.9	239.9	74.1	146.2	149.8	60.0	133.5	62.5	40.0	21.0	45.2

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